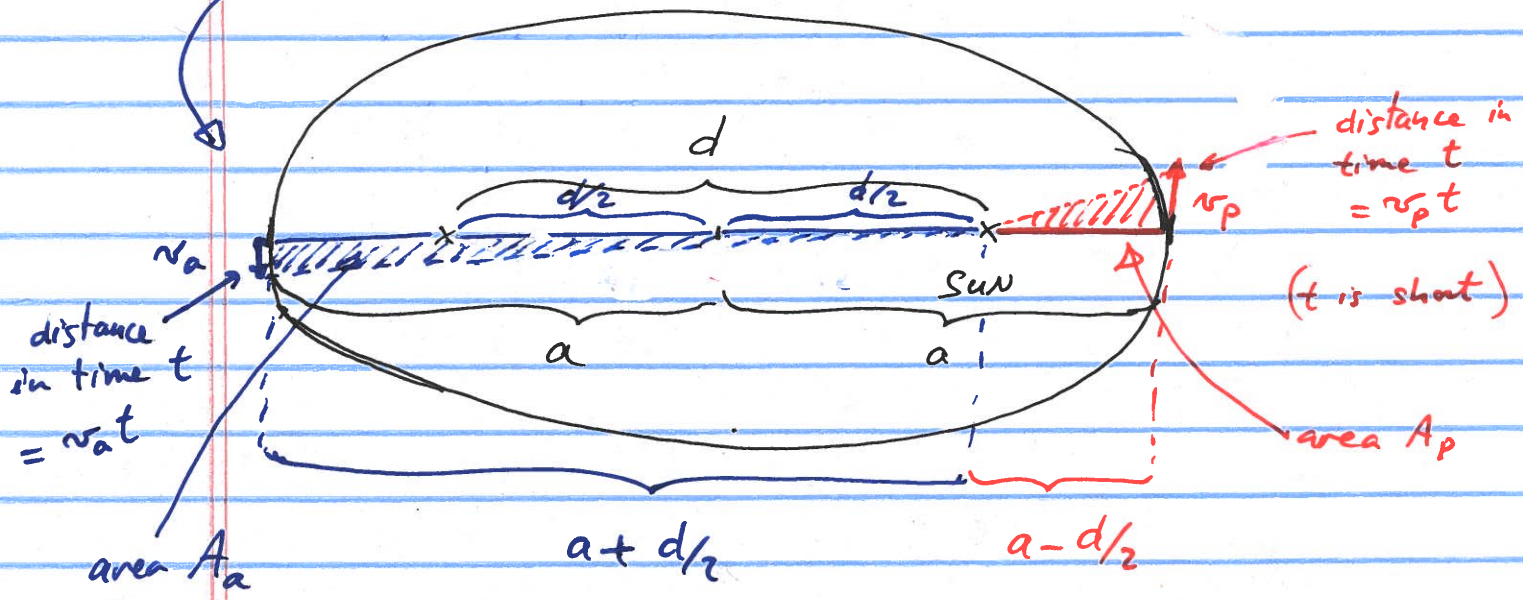
Wednesday, August 26, 2020

Speeds at aphelion and perihelion

(far, slow) (near fast)



Area of the aphelion triangle

$$A_a = \frac{(v_a t)(a + d/2)}{2}$$

Area of the perihelion triangle

$$A_p = \frac{(v_p t)(a - d/2)}{2}$$

Kepler's 2nd Law: Area of aphelion triangle = Area of perihelion triangle

$$\Leftrightarrow \frac{(v_a t)(a + d/2)}{2} = \frac{(v_p t)(a - d/2)}{2}$$

$$\Leftrightarrow v_a (a + d/2) = v_p (a - d/2)$$

$$\Leftrightarrow \quad \frac{v_p}{v_a} = \frac{a + d/2}{a - d/2} = \frac{2a + d}{2a - d}$$

multiply by $\frac{2}{2}$

divide numerator & denominator by $2a$

$$\Leftrightarrow \quad \frac{v_p}{v_a} = \frac{\frac{2a}{2a} + \frac{d}{2a}}{\frac{2a}{2a} - \frac{d}{2a}} = \frac{1 + \varepsilon}{1 - \varepsilon}$$

$$\Leftrightarrow \quad \boxed{\frac{v_p}{v_a} = \frac{1 + \varepsilon}{1 - \varepsilon}}$$

Example 1: Earth with $\varepsilon = 0.02$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.02}{1 - 0.02} \simeq 1.04$$

Earth's orbital speed varies by 4%

Example 2: Mars with $\varepsilon = 0.09$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.09}{1 - 0.09} \simeq 1.2$$

Mars's orbital speed varies by 20%

Example #3 : Mercury with $\varepsilon = 0.21$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1+0.21}{1-0.21} \simeq 1.53$$

Mercury's orbital speed varies by 53%