Friday, August 28, 2020

Kepler's laws : Applicability

Applies everywhere

1st Law: All orbits are ellipses, circles, parabolas, or hyperbolas

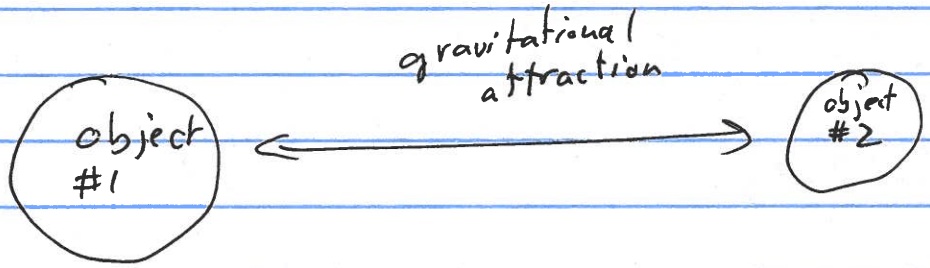
2nd Law: Law of equal areas

3rd Law: $T^2 = a^3$

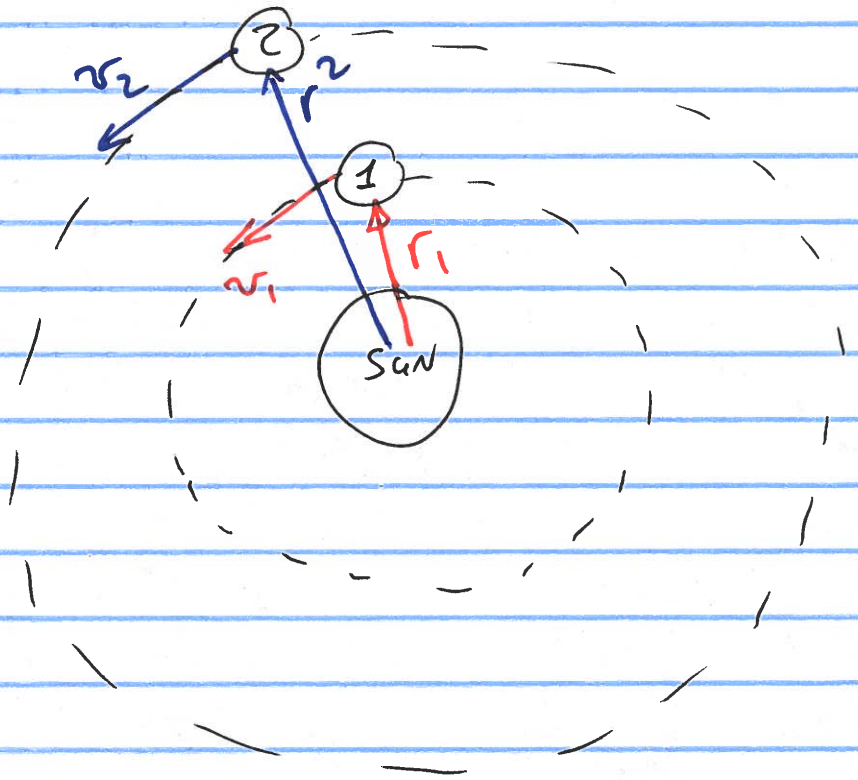
T = orbital period in Earth years

a = semimajor axis in AU.

It "should" apply everywhere, but our units only make valid for Solar System.



Orbital Speed vs. Orbital distance



Assume circular orbits for simplicity
[good approximation for our solar system]

↳ $r_1 = a_1$ | radius = semimajor axis
$r_2 = a_2$

Let's calculate velocity/speed

orbital circumference $= 2\pi r$

orbital period $= T = \frac{2\pi r}{v}$ (1) $\Leftrightarrow v = \frac{2\pi r}{T}$

orbital speed

Kepler's 3rd Law: $T^2 = a^3 = r^3$

$$\Leftrightarrow T = r^{3/2} \quad (2)$$

Combine equations (1) & (2): $\frac{2\pi r}{v} = r^{3/2}$

$$\Rightarrow v = \frac{2\pi r}{r^{3/2}} \quad (\Leftrightarrow) \quad v = \frac{2\pi}{r^{1/2}}$$

$$\Leftrightarrow v = \frac{2\pi}{\sqrt{r}}$$

units of AU/yr

units of AU

Conclusion: The further out a planet is, the slower its speed.