

Friday, October 30, 2020

Example: Luminosity of a white Dwarf (just born)
[in the Butterfly Nebula
NGC 6302]

$$T = 200,000 \text{ K} = 2 \times 10^5 \text{ K}$$

$$M = 0.64 M_{\text{sun}}$$

↳ According to white dwarf radius vs. mass plot

$$M = 0.64 M_{\text{sun}} \text{ corresponds to } R = 0.012 R_{\text{sun}}$$

$$\text{i.e. } R(0.64) \approx 0.012$$

(Wien's law) peak wavelength emission: $\lambda_{\text{max}} = \frac{2.9 \times 10^6}{200,000} = 14.5 \text{ nm}$

$$\Rightarrow \lambda_{\text{max}} = 14.5 \text{ nm}$$

This wavelength is
in the extreme
ultraviolet.
(EUV) ↑

Stefan-Boltzmann law:

$$\text{surface intensity} = \sigma T^4$$

$$= \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \right) (2 \times 10^5 \text{ K})^4$$

$$= 9.07 \times 10^{13} \approx 9 \times 10^{13} \text{ W/m}^2$$

this light
will ionize
any atom/molecule

$$\Rightarrow \text{Surface intensity} = 9 \times 10^{13} \text{ W/m}^2$$

↑
note: for our Sun
surface intensity = $6.4 \times 10^7 \text{ W/m}^2$
~million times
more intense than Sun's surface!

Surface area of white dwarf:

$$\text{Surface Area} = 4\pi R^2 = 4(3.1415926)(8.35 \times 10^7)^2 \\ = 8.77 \times 10^{14} \text{ m}^2$$

$$R = 0.12 R_{\text{sun}} = (0.12) \left(\frac{6.96 \times 10^5 \text{ km}}{1000} \right) \\ = 8.35 \times 10^6 \text{ m}$$

$$\text{Luminosity} = \text{total output power} = \underbrace{\text{surface intensity}}_{\text{W/m}^2} \times \underbrace{\text{Surface area}}_{\text{m}^2} \\ = (9.10^{13} \text{ W/m}^2) (8.77 \times 10^{14} \text{ m}^2) \\ = 7.75 \times 10^{28} \text{ W} \\ \approx 8 \times 10^{28} \text{ W}$$

$$\text{Luminosity of white dwarf} = 8 \times 10^{28} \text{ W} \approx 200 L_{\text{sun}} \\ \text{(just born)}$$

note: luminosity of Sun $\approx 4 \times 10^{26} \text{ W} = L_{\text{sun}}$
(200 times less luminous)

