

Problem Set #7

1. Solar consumption

- Calculate the equivalent mass loss of the Sun (due to emitted power) over the course of 1 second (kg/s) and over the course of 1 year (kg/yr). The output power of the Sun is 3.9×10^{26} W.
- Assuming that the fusion only happens within the core of the Sun ($R_{core} = 0.2R_{Sun}$), calculate the average power generated per unit volume for solar fusion (answer in W/m^3).
- The Sun is expected to “burn” about 10% of its total mass during its primary evolution (i.e. %10 of the mass participates in a fusion). Of this 10%, only a small fraction is actually converted to energy (you will have to look this up in the notes). Assuming that the Sun started with a mass of $M_{Sun} = 1.99 \times 10^{30}$ kg, calculate the lifetime of the Sun during its primary evolution.

2. Binary star masses

Sirius is actually a binary star system: Sirius A (the “dog star” with mass M_A) is the bright star you see in the sky, and Sirius B (“the pup” with mass M_B) is a very faint white dwarf star. These two stars orbit each other, and, in this problem, you will determine the masses of these two stars from observational parameters of their orbital motion.

a) Total mass: The two stars orbit each other with a period of $T = 50.1$ years and an average distance from each other of $a = 19.8$ AU (i.e. semimajor axis). Calculate the total mass of the Sirius binary star system $M_{total} = M_A + M_B$ in kilograms.

Hint: Consider one of the versions of Kepler’s laws.

b) Mass ratio: Telescope observations indicate that the center-of-mass of the star system is located 33% of the way from Sirius A on the line segment that connects the two stars. In other words, if we call a_1 the distance of M_A from the center-of-mass and a_2 the distance of M_B from the center-of-mass, then $a_1 = 0.33a$ and $a_2 = 0.67a$, with $a_1 + a_2 = a$.

Which star is heavier? Sirius A or Sirius B? Calculate the mass ratio M_A/M_B .

Hint: Review Lecture 7 (part A), which discusses center-of-mass physics.

c) Individual masses: You know $M_A + M_B$ and M_A/M_B . Determine M_A and M_B in kilograms and in units of solar masses (i.e. M_{Sun}).

3. Detecting Jupiter from an alien star system

Aliens on an exo-planet in a distant star system, which happens to lie in our ecliptic plane, direct their planet-hunting telescope at our Solar System to see if it has any planets. In your calculations, you can assume that the disk of the Sun is uniformly bright, even if the aliens’ telescope cannot resolve the Sun’s disk (i.e. it is just a star-like point).

- Calculate the % change in the Sun’s brightness (as seen by the aliens) due to Jupiter passing in front of the Sun. Does the Sun get brighter or dimmer when Jupiter “eclipses” the Sun?
- How long does the “eclipse” last? (*Hint: first figure out the orbital speed of Jupiter.*)