Wednesday, November 4, 2020

Example: Rotation frequency of a Sun-based white dwarf .... and "neutron star".

major assumption: no mass loss

note: $f_{sun} = 4.63 \times 10^{-7}$ Hz

$T_{sun} \approx 25$ days $= 2.16 \times 10^6$ s

the Sun cannot produce a neutron star, but this a "what if" exercise.

According to the Radius vs. mass plot, a 1 solar mass white dwarf has a radius of

$$R_{wd} = 0.0078\ R_{sun} = 5400\ km$$

For the neutron star we will use $R_{ns} = 14\ km$

$$= \frac{14}{6.96 \times 10^5} R_{sun}$$

$$\simeq 2 \times 10^{-5}\ R_{sun}$$

when the Sun contracts into a white dwarf (no mass loss) angular momentum is conserved.

$L = mvr$ for a particle going in a circle

$$v = \frac{2\pi r}{T} = 2\pi r f$$

(T: period; f: frequency)

$\Rightarrow L = 2\pi r^2 f m$

proportional

for a rotating sphere $L \propto 2\pi r^2 f m$

or $L = \alpha\, 2\pi r^2 f m$

proportionality constant that accounts for geometry of sphere

If angular momentum is conserved, then

$$L_{sun} = \alpha\, 2\pi\, r_{sun}^2\, f_{sun}\, m$$
$$L_{wd} = \alpha\, 2\pi\, r_{wd}^2\, f_{wd}\, m$$
$$L_{ns} = \alpha\, 2\pi\, r_{ns}^2\, f_{ns}\, m$$

with $L_{sun} = L_{wd} = L_{ns}$ (m remains unchanged by assumption)

Thus $\frac{L_{sun}}{L_{wd}} = 1 = \frac{\alpha\, 2\pi\, r_{sun}^2 f_{sun}\, m}{\alpha\, 2\pi\, r_{wd}^2 f_{wd}\, m} \Leftrightarrow$ $\boxed{f_{wd} = \left(\frac{r_{sun}}{r_{wd}}\right)^2 f_{sun}}$

$$\Rightarrow f_{wd} = \left(\frac{1}{0.0078}\right)^2 f_{sun} = (16437)(4.63\times10^{-7}) = 0.00761 \text{ Hz}$$

$\Rightarrow$ $\boxed{f_{wd} = 0.0076 \text{ Hz} \Leftrightarrow T_{wd} = 131\text{ s} = 2 \text{ minutes } 11 \text{ seconds}}$

neutron star: $\boxed{f_{ns} = \left(\frac{r_{sun}}{r_{ns}}\right)^2 f_{sun}} = \left(\frac{1}{2\times10^{-5}}\right)^2 f_{sun} = (2.5\times10^{9})(4.63\times10^{-7}) = 1157 \text{ Hz}$

$\Rightarrow$ $\boxed{f_{ns} = 1157 \text{ Hz} \Leftrightarrow T_{ns} = 0.86 \text{ ms} = 0.86\times10^{-3}\text{ s}}$