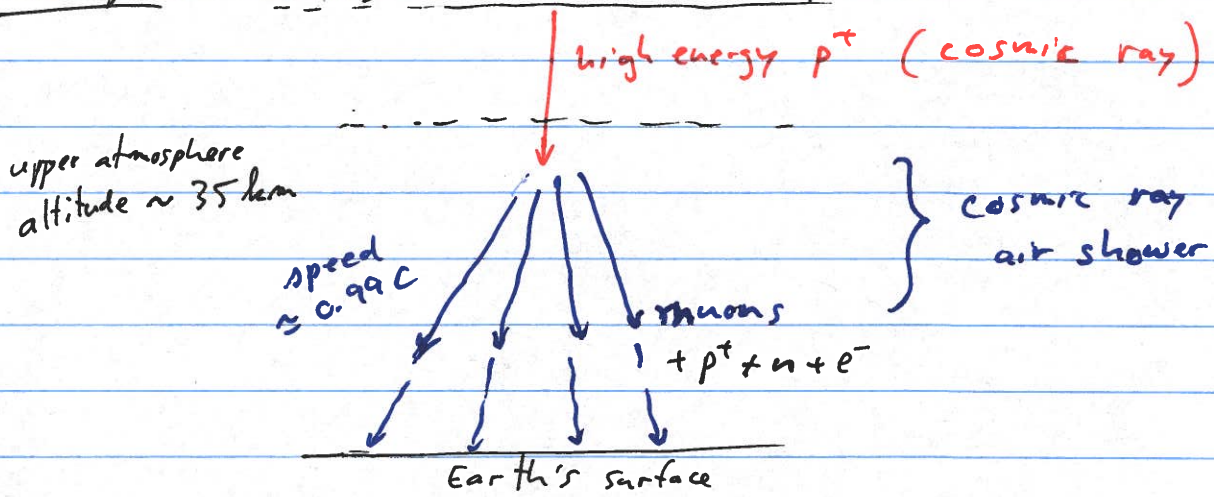
Friday, November 6, 2020

Example: Muons from cosmic rays



A muon is a "heavy electron" with the same charge as an electron but with 207 times the mass of the electron.

Muons are unstable and have a lifetime of 2.2 μs at rest.

Galilean "naive" travel distance

$$\Delta z = \Delta t \times v = (2.2 \times 10^{-6}\ s)(0.99 \times 3 \times 10^8\ m/s)$$

$0.99 \times 3 \times 10^8\ m/s = 2.97 \times 10^8\ m/s$

$$= 653\ m$$

wrong

Experimental fact: cosmic ray muons are observed in large numbers on Earth surface

Special Relativity calculation of travel distance

Muons "live" for 2.2 μs = $\Delta t'$ in their rest frame

In our frame (Earth's frame), the muon lifetime experiences time dilation

$$\Delta t = \gamma \Delta t' \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and } v = 0.99c$$

Δt: in Earth's frame

$$= 50.3 \times 2.2\,\mu s$$
$$= 110.5\,\mu s$$
$$= 110.5 \times 10^{-6}\,s$$

$$= \frac{1}{\sqrt{1-\left(\frac{0.99c}{c}\right)^2}}$$

$$= \frac{1}{\sqrt{1-0.9801}}$$

$$= 50.3$$

In Earth's frame, the muons lifetime, i.e. travel time is

$$\boxed{\Delta t \simeq 110.5 \times 10^{-6}\,s} \simeq 110 \times 10^{-6}\,s$$

Vertical travel distance of muons: $\Delta z = \Delta t \times v$ ($v = 0.99c$)

$$= (110 \times 10^{-6})(2.97 \times 10^{8}\,m/s)$$
$$= 32834\,m$$
$$\simeq 32.8\,km$$

$$\Rightarrow \boxed{\Delta z \simeq 33\,km}$$

$\Rightarrow$ many muons should make it to the Earth's surface.