Today's Topics

Friday, November 6, 2020 (Week 11, lecture 32) – Chapter 24.

A. Einstein's Theory of Relativity.

B. Special Relativity.

C. Length contraction.

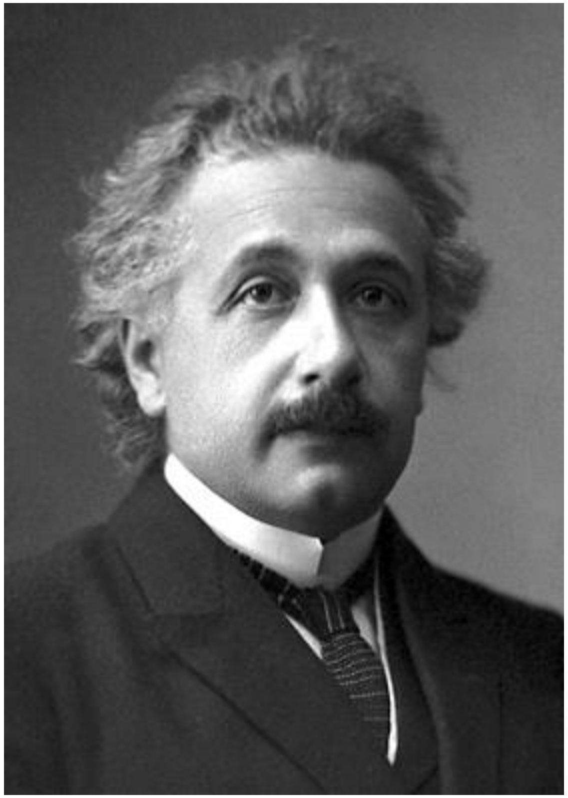
D. Time dilation.

E. General Relativity.

Einstein's Theory of Relativity

1905: Annus Mirabilis

- Brownian motion (motion of atoms in a gas).
- Photo-electric effect (discovery of the photon, $E = hf$)
- **Special theory of relativity**.
  - → Major revision of Galilean relativity.
  - → Equivalence of energy and matter: $E = mc^2$



Albert Einstein, 1921.
(1879-1955)

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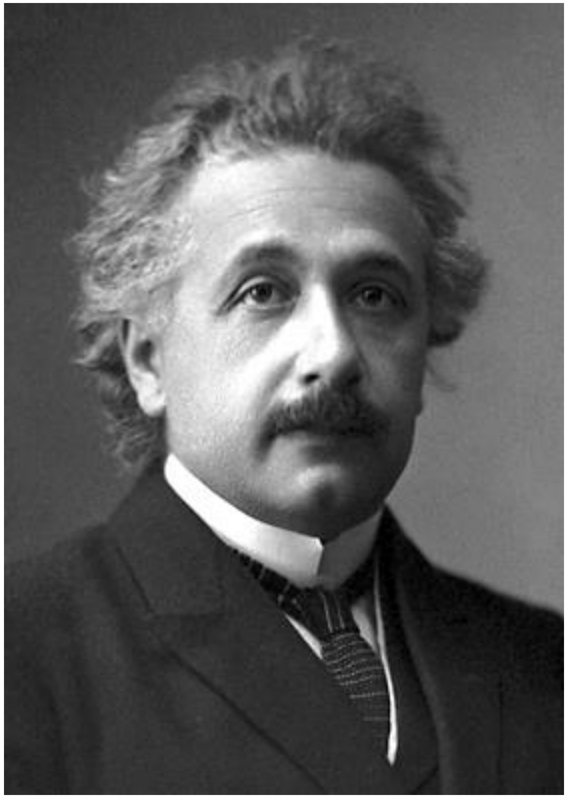
1907-15: General Relativity

Theory of relativity applied to gravity.
→ gravity = curved space-time.



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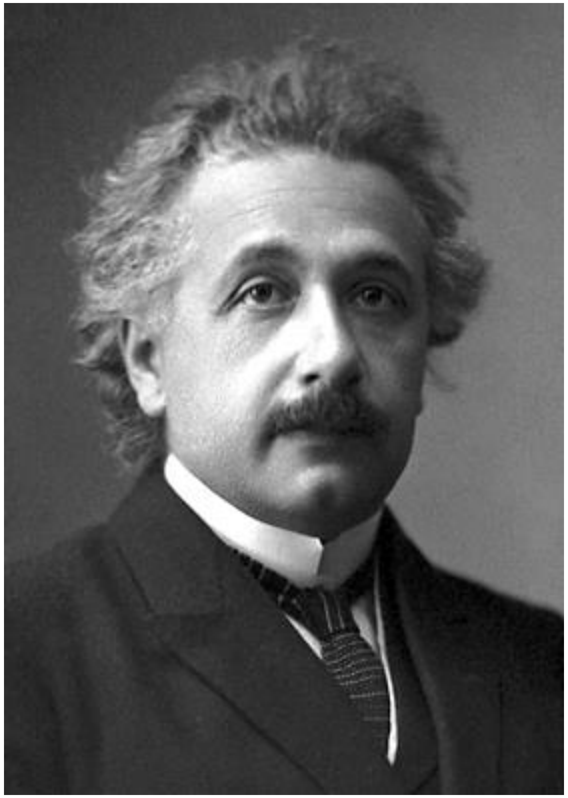

Albert Einstein, 1921.
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1921: Nobel Prize for photo-electric effect.

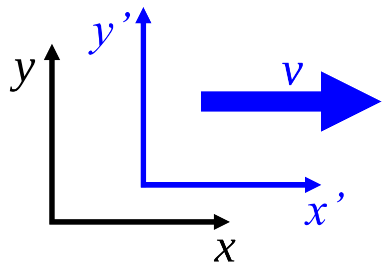
1924: Bose-Einstein Condensation
Predicts the existence of a new type of quantum matter.
→ Builds on the work of Satyendra Bose.
→ First observed in 1995
→ There is a BEC in the basement of Small Hall (room # 069).

Inertial Frames (Galileo & Einstein)

Inertial Frame

Coordinate system at constant velocity in a rest frame.

think of it as a box



Rest Frame

A coordinate system that is not moving.

Note: a rest frame is an inertial frame.

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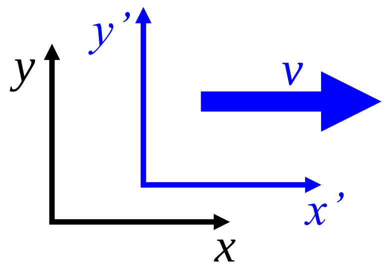

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Important

- **You cannot tell if you are moving** based on local measurements inside your inertial reference frame (the frame attached to you).
- If you are **accelerating/decelerating**, then you can tell based on local measurements (i.e. there is a force on you that you can measure, $F = ma$).

Special Relativity (Einstein)

Principle of Relativity

The laws of physics are the same in all inertial reference frames.

Corollary #1

You cannot tell if you are moving (based on local measurements) in an inertial frame.

Corollary #2: Universal speed of light

The speed of light in vacuum is the same in all inertial frames, regardless of the motion of the source.

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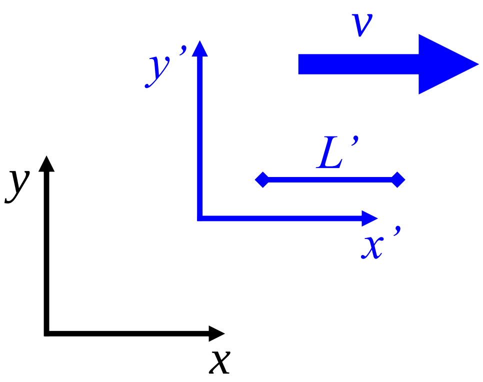
→ Length contraction & time dilation

Special Relativity
Length Contraction

In the x'-y' inertial frame

Consider a rod of length $L' = L_0$, as measured in the x'-y' inertial frame (i.e. the rest frame of the rod).

Note: The rod is aligned with the axis of motion along x'.



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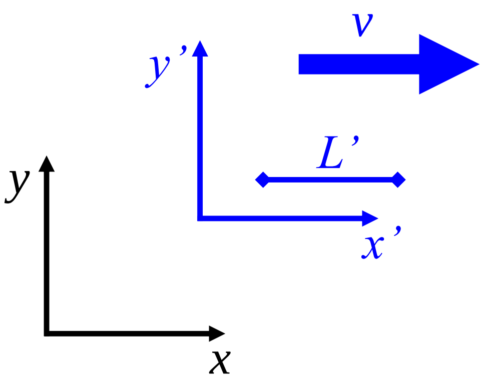


In the x-y inertial frame

If you measure the length of the rod, then you will get a shorter length: $L = \frac{L_0}{\gamma}$.

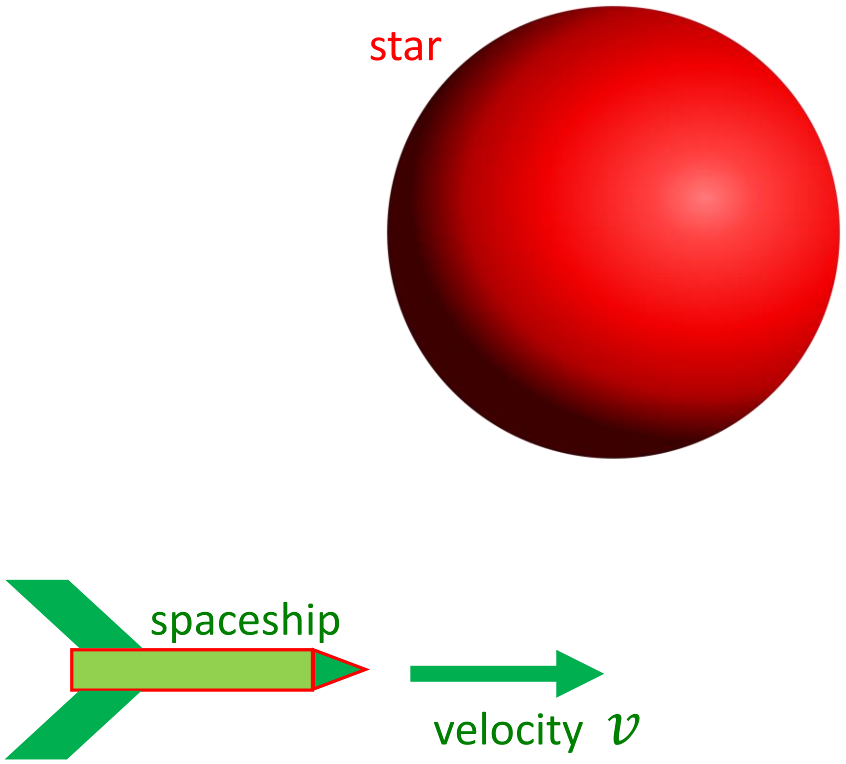
Gamma factor: $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$\gamma \geq 1$

Note: the length contraction is only along the axis of motion. Along axes perpendicular to the motion, there is no change in length.

Length Contraction: Example

Consider a spaceship travelling past a spherical star at 90% of the speed of light.



Question: What is the shape of the star in the frame of the spaceship?

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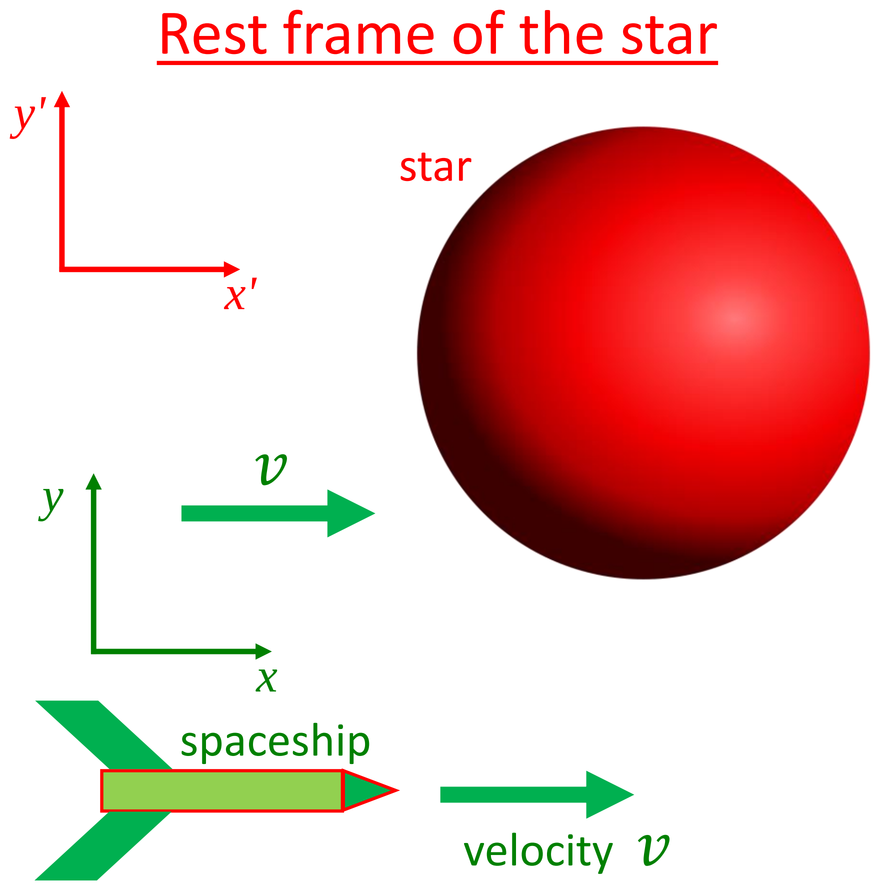

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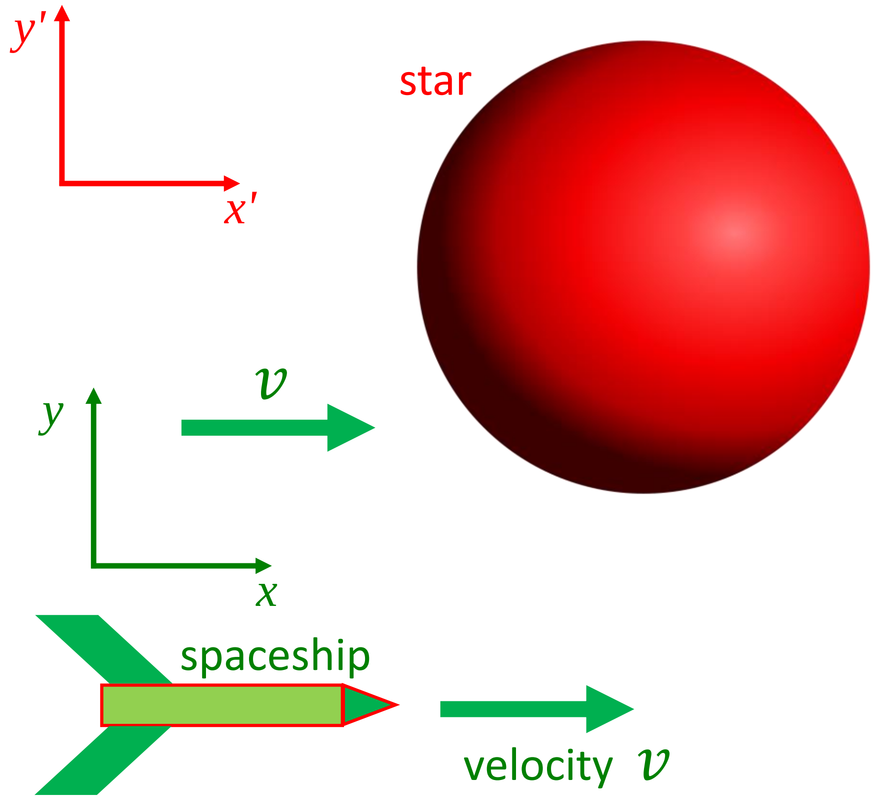

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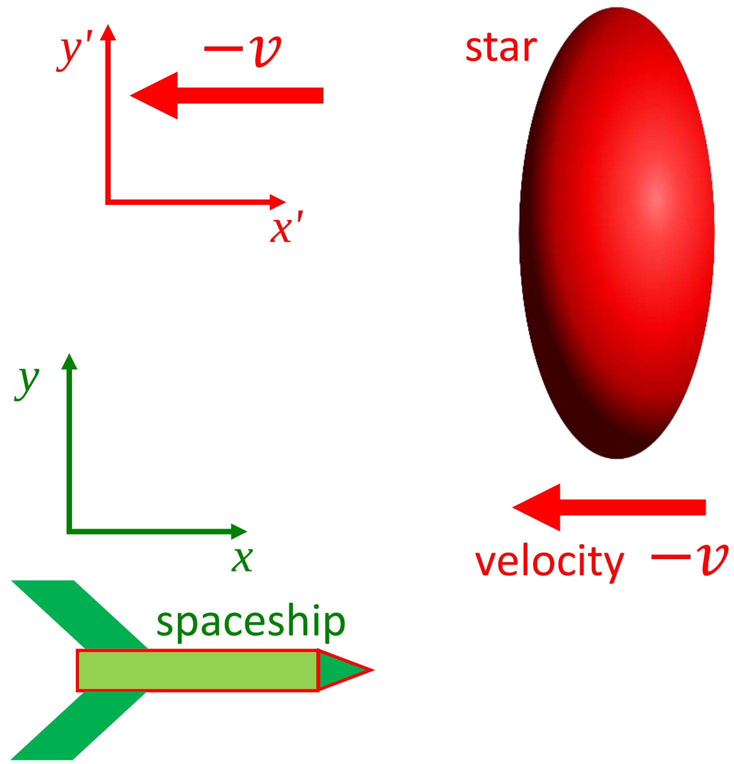


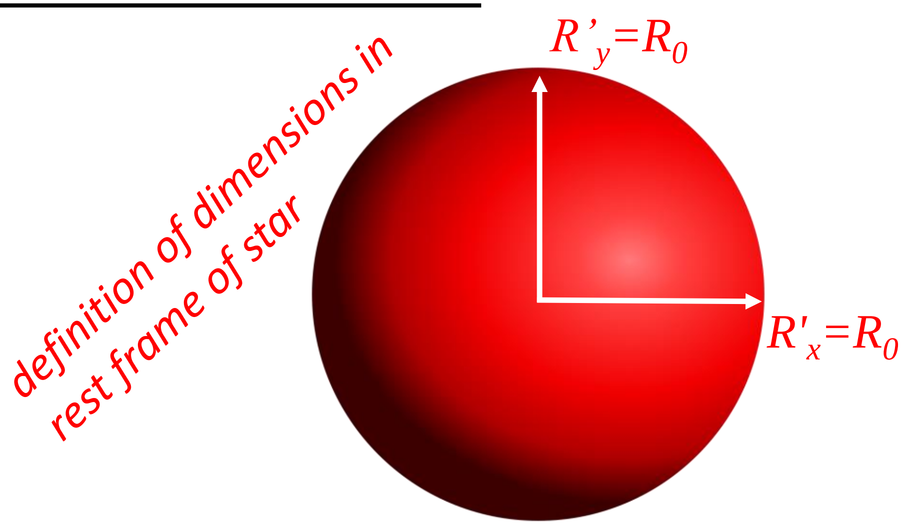
Answer: The star appears/is compressed along the axis of travel.

The transverse directions are unaffected.

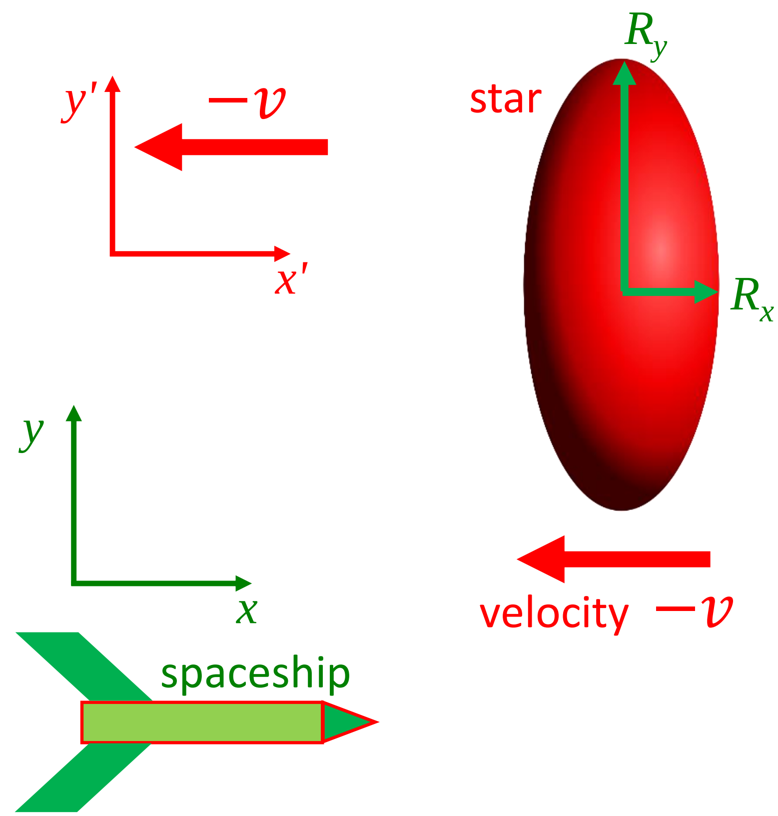
Length Contraction: Example

Consider a spaceship travelling past a spherical star at 90% of the speed of light.

Quantitative answer



Rest frame of the spaceship



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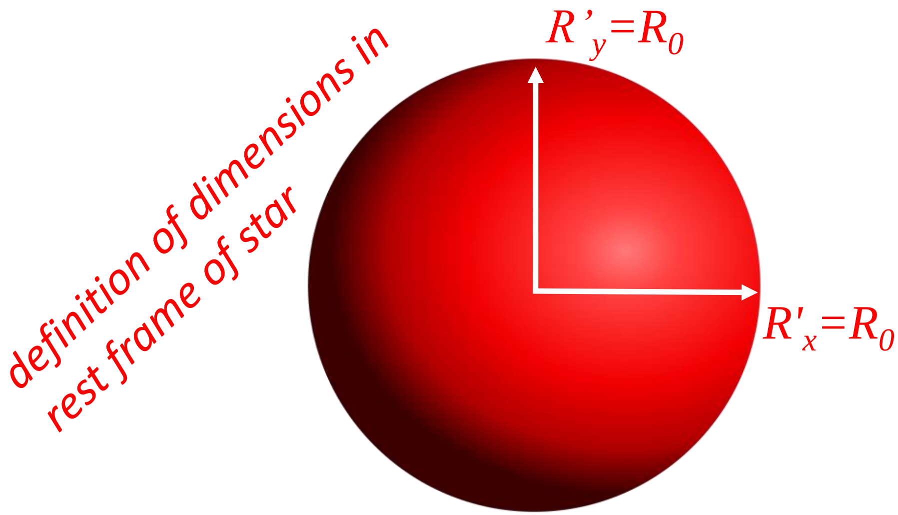
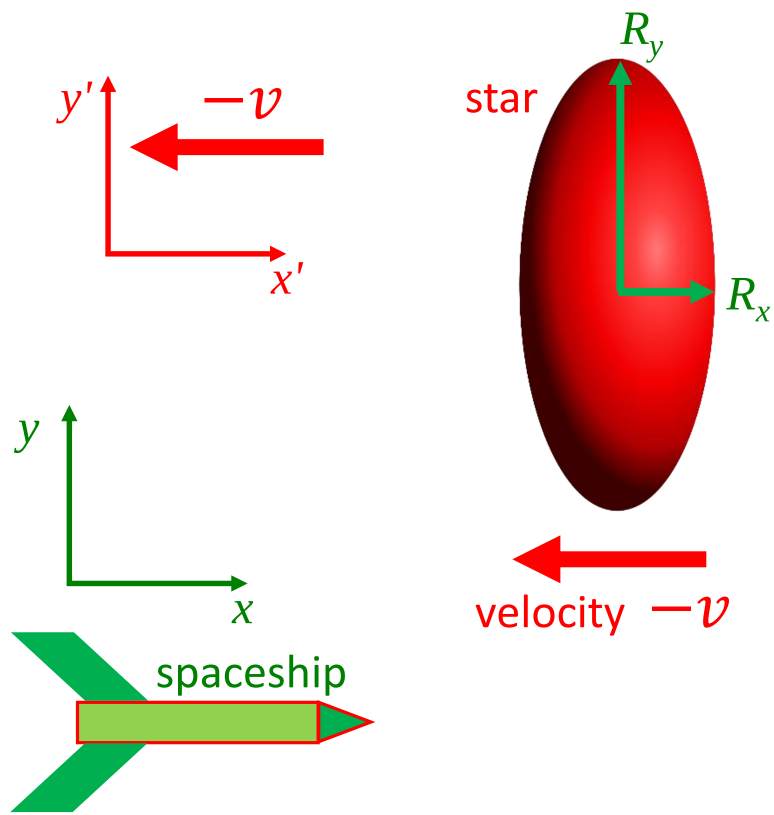


In the rest frame of the spaceship, we have

$$R_x = \frac{R_0}{\gamma} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Rest frame of the spaceship



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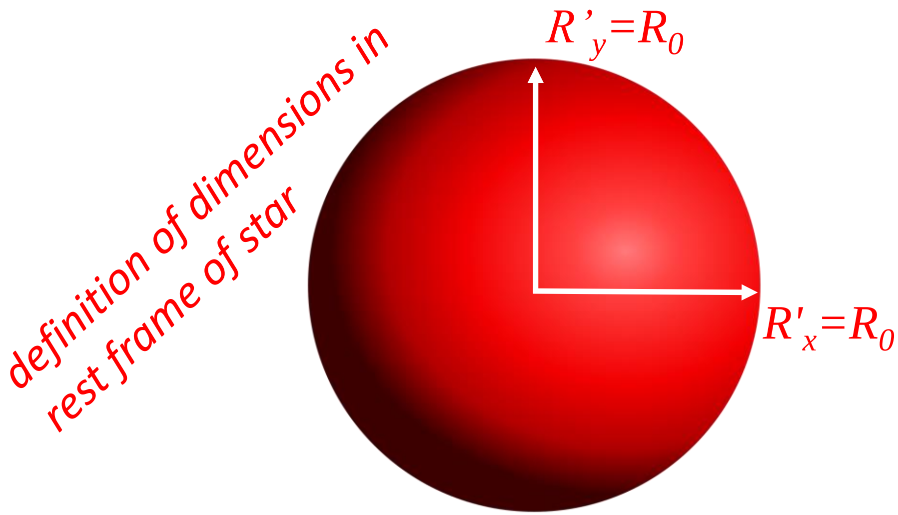
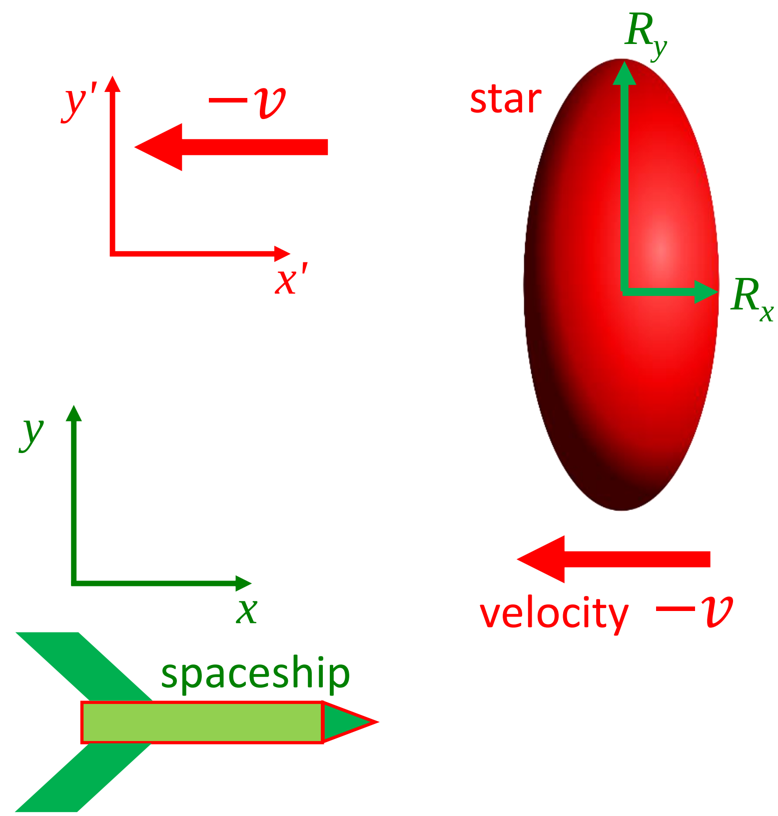


In the rest frame of the spaceship, we have

$$R_x = \frac{R_0}{\gamma} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(-0.9c)^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1-0.81}} = 2.29$$

Rest frame of the spaceship



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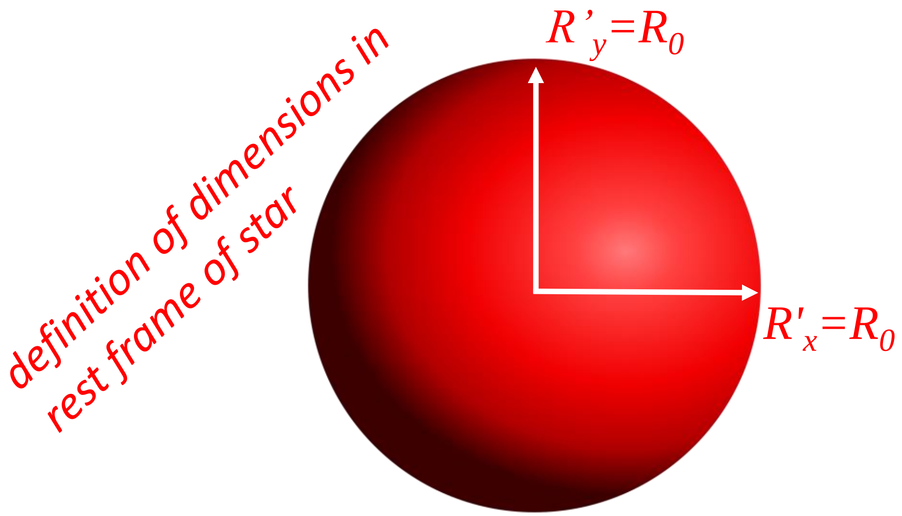

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Thus $R_x = \frac{R_0}{2.29} = 0.43 R_0$

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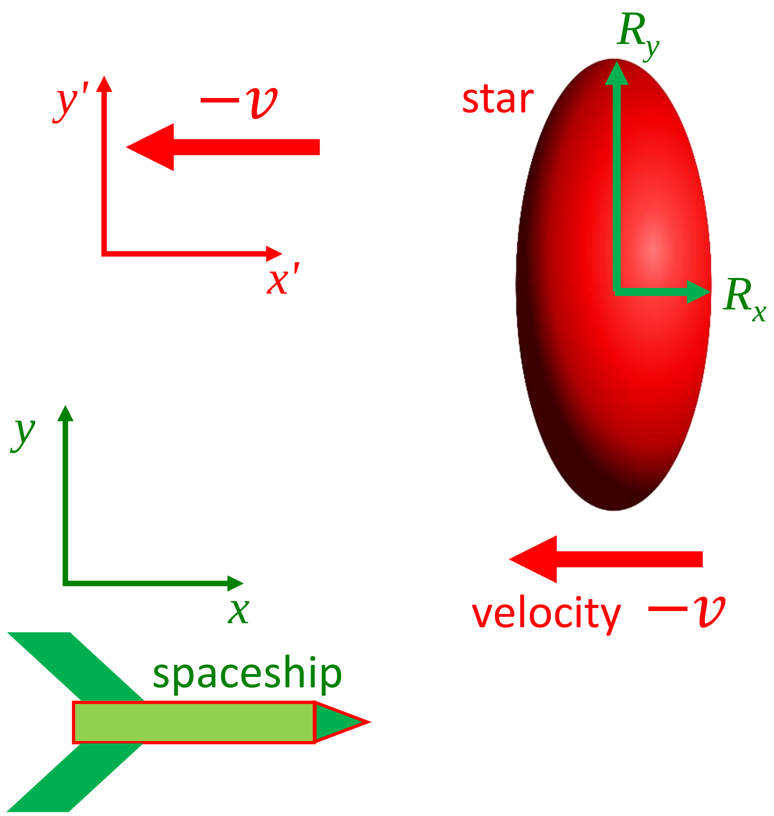

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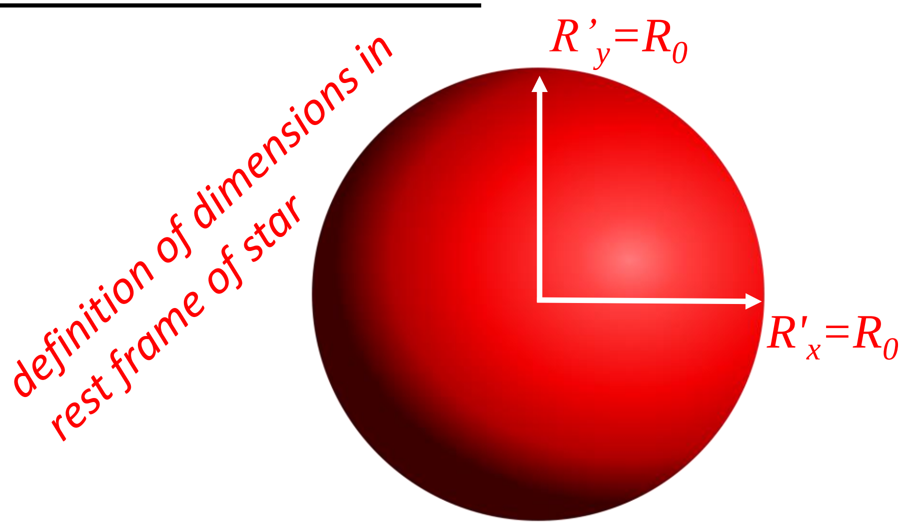

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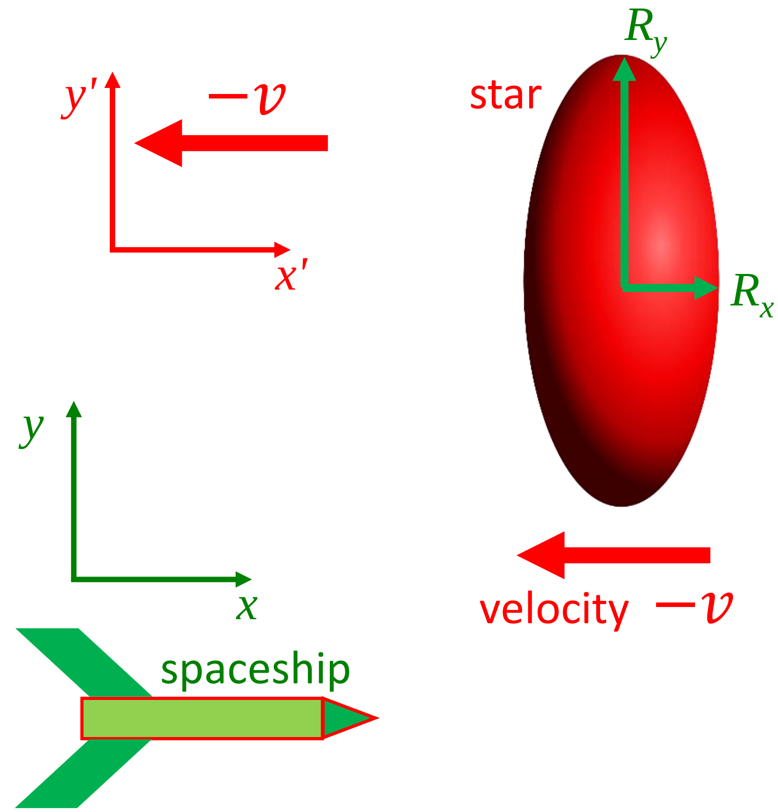
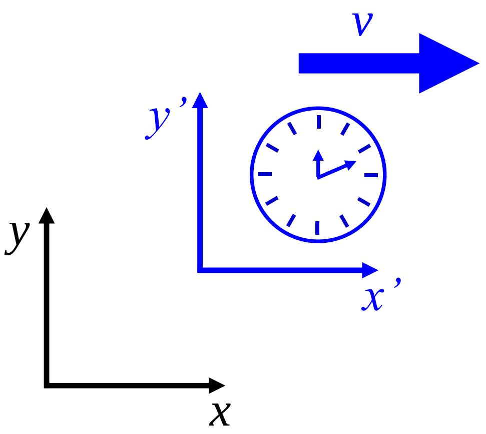


Answer: The star appears/is compressed to 43% of its original size along the direction of travel.
The transverse directions are unaffected.

Special Relativity
Time Dilation

In the x'-y' inertial frame

Consider a clock at rest in the x'-y' inertial frame that measures a time interval of $\Delta T' = T_0$, i.e. the time for the big clock hand to go from noon to the 2 o'clock position (10 minutes).



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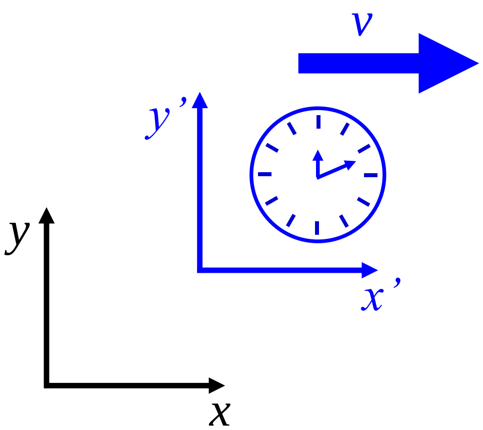

In the x-y inertial frame

If you measure the same elapsed time (with your own timepiece) from the x-y inertial frame, i.e. as the clock flies past you, then you will measure a longer elapsed time: $T = \gamma T_0$.

Time Dilation Example
The Twin Paradox

- Twin A travels to a distant star at a velocity of $v = 0.9c$ and then returns also at a velocity $v = 0.9c$, while twin B remains on Earth.
- Twin A measures a travel time of 10 years (according to twin A's clock) to get to the star, and then 10 years to return to Earth.

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How much older is twin A, when twin A returns to Earth?

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Question 1

How much older is twin A, when twin A returns to Earth?

Answer 1

Since we are using twin A's clock, we know that

$$\Delta T' = T_0 = 2 \times 10\, years = 20\, years$$

Twin A has aged 20 years (in the physics-biology sense).

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Question 2

How much older is twin B, when twin A returns to Earth?

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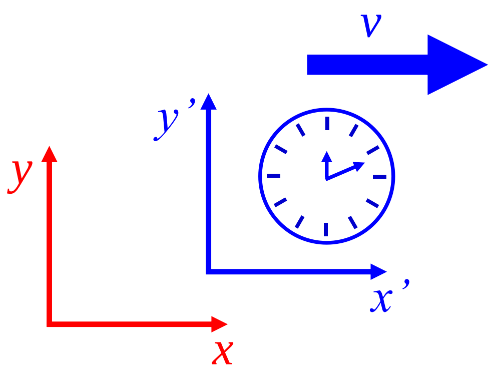

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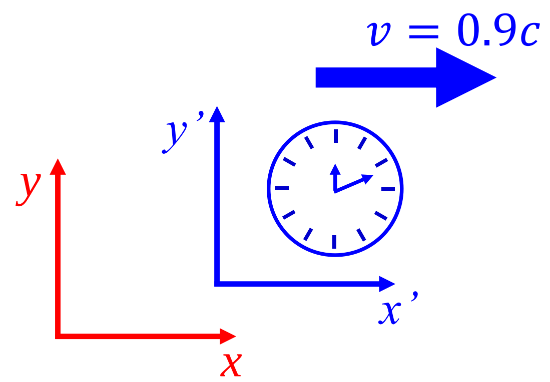

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Answer 2

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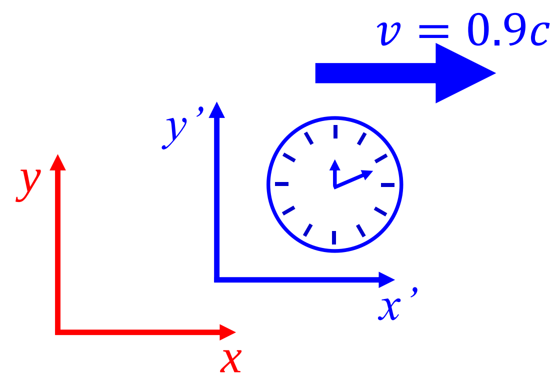

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$$\Delta T = \gamma \Delta T' = \gamma T_0 = 2.29 \times 20\, years = 45.8\, years$$

with $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2.29$

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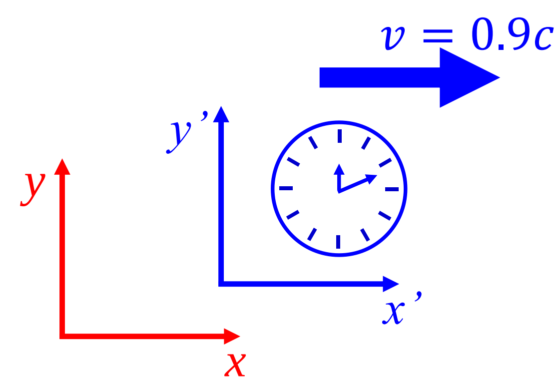

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Twin B has aged 45.8 years while remaining on Earth !!!

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Answer 3

Twin A must accelerate and decelerate, so twin A is briefly in a **non-inertial frame**. The motions of twin A & twin B are not symmetric.

General Relativity

Equivalence Principle

A coordinate system that is falling freely in a gravitational field is (equivalent to) an inertial frame.

Corollary

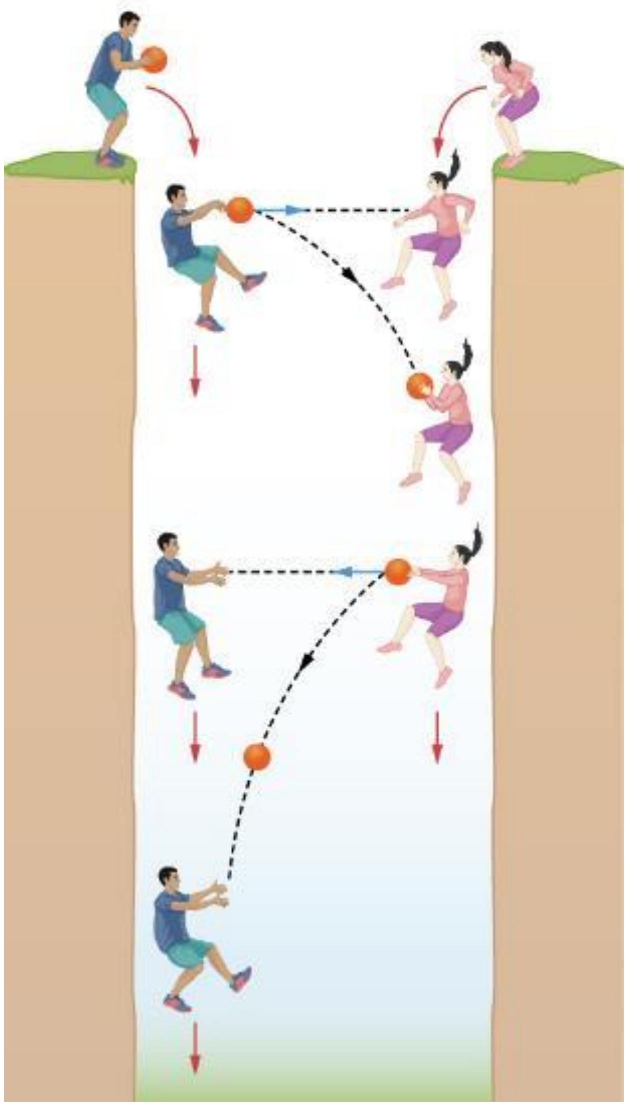
You cannot tell if you are at rest in a non-gravitational field (i.e. in a standard inertial frame) or freely falling under gravity based on local measurements.

Equivalence Principle

You cannot tell if you are at rest in free space (i.e. in a standard inertial frame) or freely falling under gravity based in based on local measurements.

Example

- Two people play catch as they descend into a bottomless abyss.
- Since the people and ball all fall at the same speed, it appears to them that they can play catch by throwing the ball in a straight line between them.
- Within their frame of reference, there appears to be no gravity.

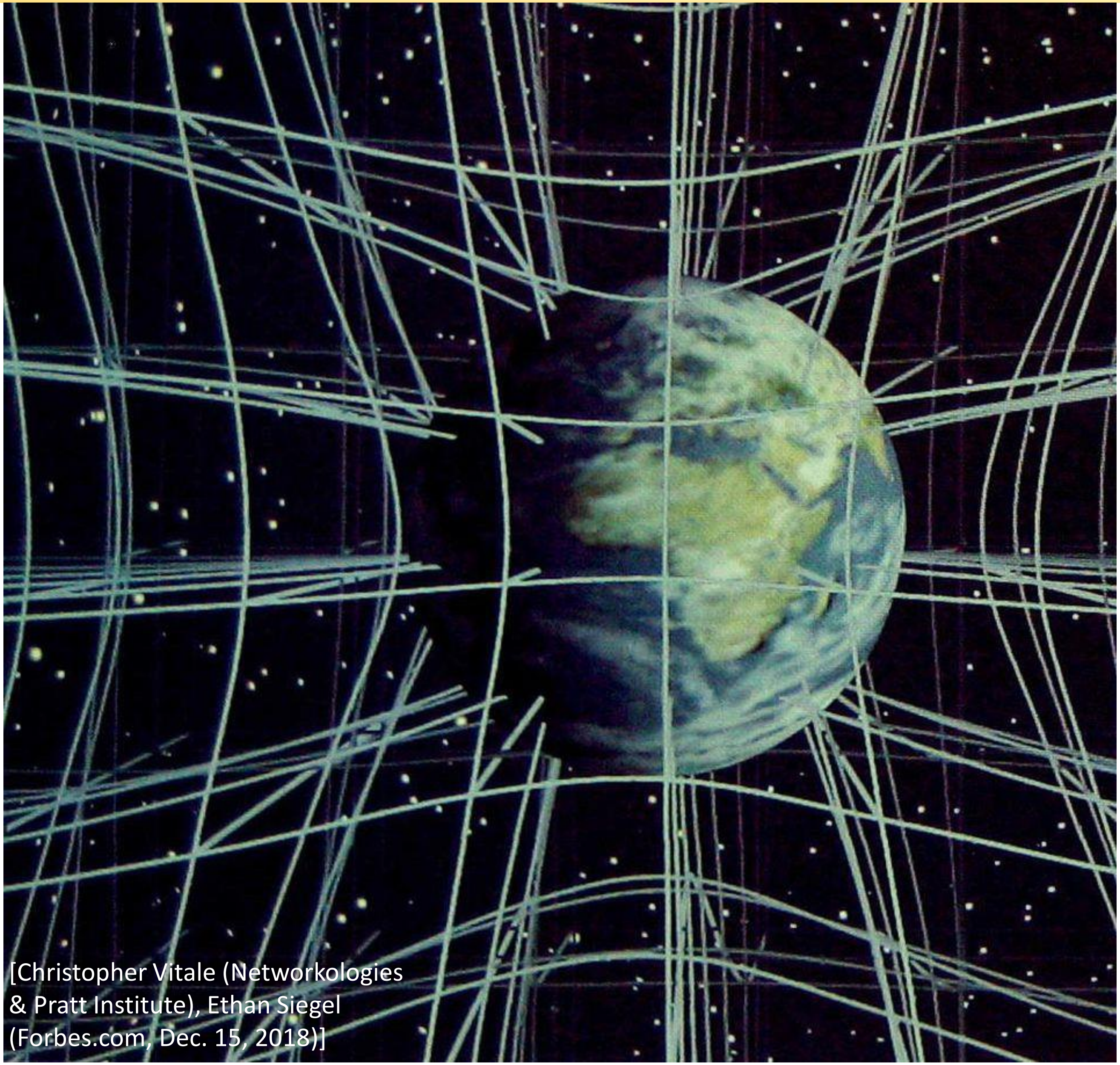


[OpenStax: Astronomy]

Equivalence Principle on ISS

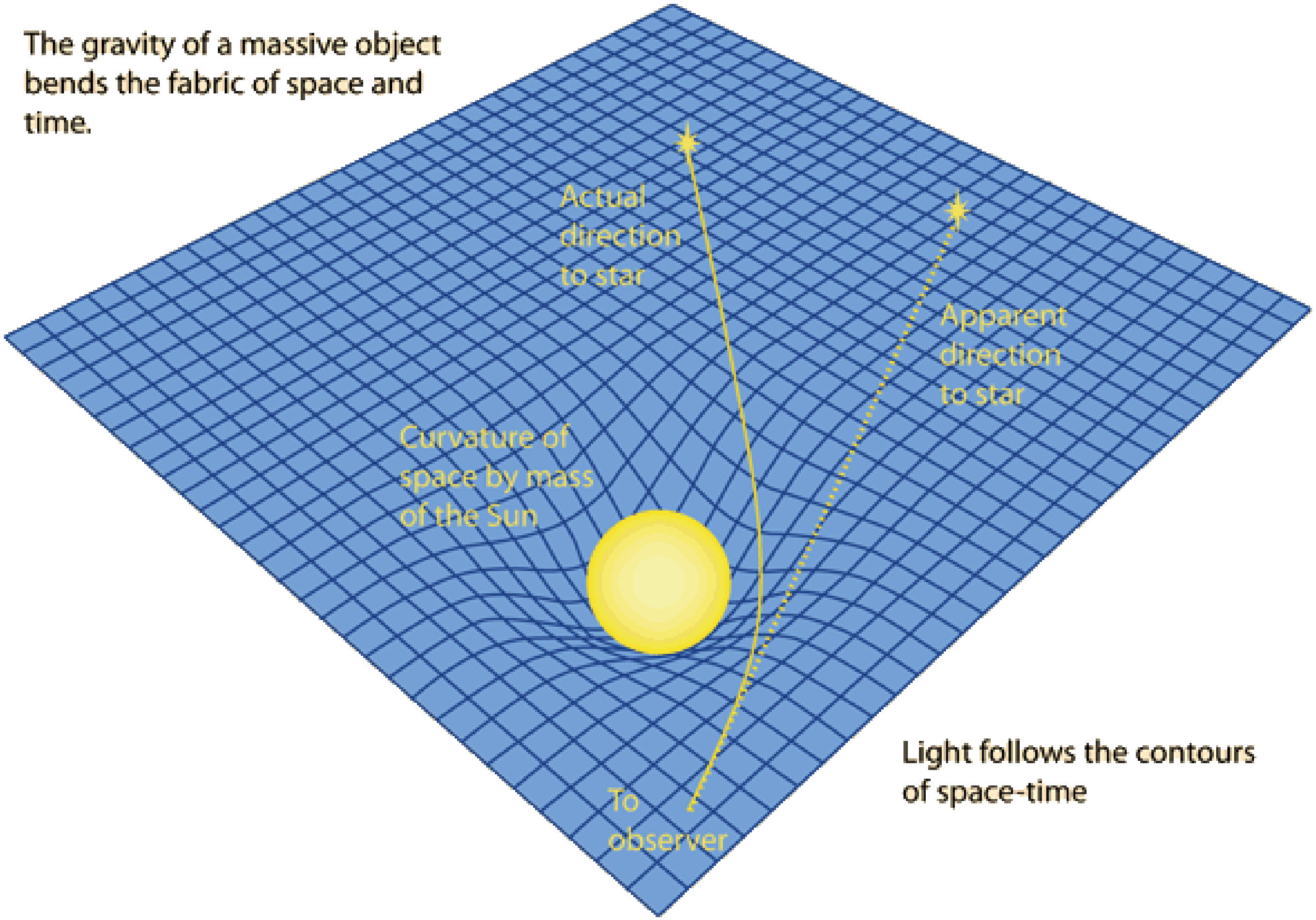
[NASA]

Curved Space-Time



[Christopher Vitale (Networkologies & Pratt Institute), Ethan Siegel (Forbes.com, Dec. 15, 2018)]

Curved Space-Time: light rays in 2D



[http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/grel.html]