

Friday, November 13, 2020

Example: Time dilation near a black hole

Q: How much slower does a clock run when it is 1 m from the event horizon of a black hole?  
 i.e. how much faster does the rest of the universe run / tick?

Black hole at center of Milky Way:  $R_S \approx 17 R_{Sun}$   
 (Sagittarius A\*)  
 $= 12 \times 10^6 \text{ km}$   
 $= 12 \times 10^9 \text{ m}$

clock 2 frequency  $= \frac{f_2}{f_1} = \sqrt{\frac{1 - \frac{R_S}{R_2}}{1 - \frac{R_S}{R_1}}}$

$R_1 \leftarrow R_1 = R_S + 1 \text{ m}$

$R_2 \rightarrow +\infty$   
 very far away

$\approx \sqrt{\frac{1}{1 - \frac{R_S}{R_1}}}$

$\approx \sqrt{\frac{1}{\frac{R_1 - R_S}{R_1}}}$

$= \sqrt{\frac{1}{\frac{R_1 - R_S}{R_1}}}$

Diagram illustrating the black hole geometry. The center is labeled 'center, singularity'. The event horizon is at radius  $R_S$ . A point is marked at  $R_1 = R_S + 1 \text{ m}$ . The region inside the event horizon is shaded and labeled 'Black hole'. A vertical axis represents the radial distance from the center, with  $R_2 \rightarrow +\infty$  indicating 'very far away'.

$$\Rightarrow \frac{f_2}{f_1} = \sqrt{\frac{R_1}{R_1 - R_s}} = \sqrt{\frac{12 \times 10^9 + 1}{1}}$$

$$R_1 - R_s = R_s + 1\text{m} - R_s = 1\text{m}$$

$$\approx \sqrt{12 \times 10^9}$$

$$= 1.095 \times 10^5$$

$$\approx 110,000$$

At 1 m from the event horizon, the rest of the universe evolves at a rate 110,000 times faster <sup>than</sup> you.

note: at 1 m from the event horizon of the M87 black hole, the rest of the universe evolves at a rate  $4.4 \times 10^6$  times

$$\left[ R_{s, \text{M87}} = 128 \text{ AU} = 1.92 \times 10^{13} \text{ m} \right]$$

4 faster than you  
4.4 million times faster