Friday, November 13, 2020

Example: Time dilation near a black hole

Q: How much slower does a clock run when it is 1 m from the event horizon of a black hole?
i.e. how much faster does the rest of the universe run / tick?

Black hole at center of Milky way : $R_s \simeq 17 \; R_{sun}$
(Sagittarius A^*)
$= 12 \times 10^6$ km
$= 12 \times 10^9$ m

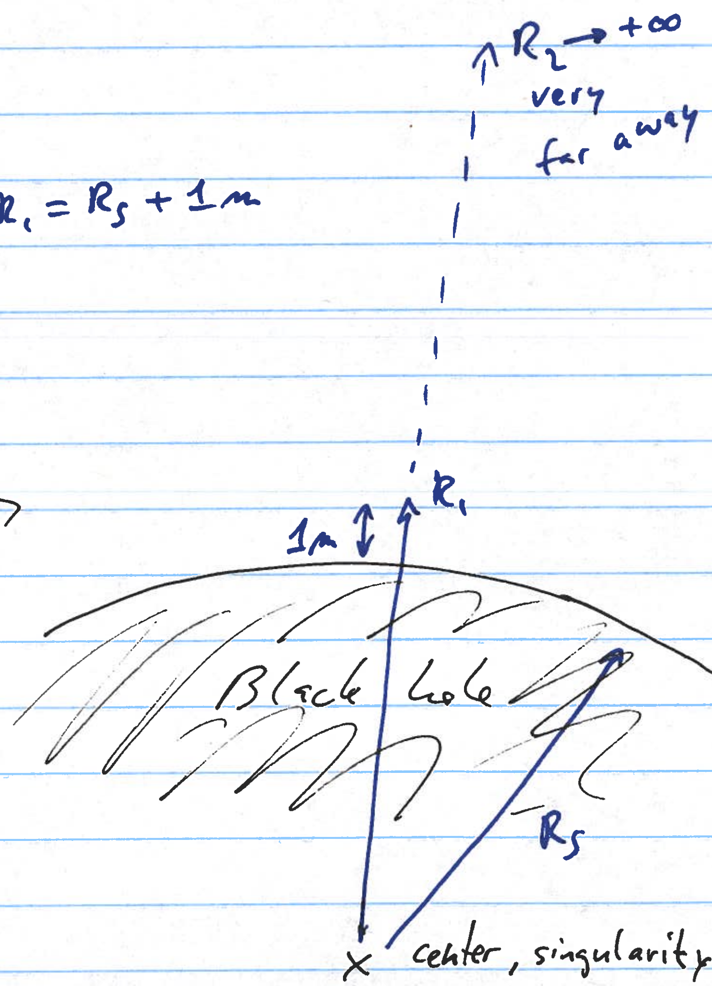
$$\frac{\text{clock 2 frequency}}{\text{clock 1 frequency}} = \frac{f_2}{f_1} = \sqrt{\frac{1 - \cancel{\frac{R_s}{R_2}}^{\,0}}{1 - \frac{R_s}{R_1}}}$$

$R_1 \leftarrow R_1 = R_s + 1\,m$

$$\simeq \sqrt{\frac{1}{1 - \frac{R_s}{R_1}}}$$

$$\simeq \sqrt{\frac{1}{\frac{R_1}{R_1} - \frac{R_s}{R_1}}}$$

$$= \sqrt{\frac{1}{\frac{R_1 - R_s}{R_1}}}$$



$$\Rightarrow \frac{f_2}{f_1} = \sqrt{\frac{R_1}{\underbrace{R_1 - R_s}_{\substack{R_s + 1m - R_s \\ = 1m}}}} = \sqrt{\frac{12 \times 10^9 + 1}{1}}$$

$$\simeq \sqrt{12 \times 10^9}$$

$$= 1.095 \times 10^5$$

$$\simeq 110,000$$

$\boxed{\text{At 1 m from the event horizon, the rest of the universe evolves at a rate 110,000 times faster than you.}}$

<u>note:</u> at 1 m from the event horizon of the M87 black hole, the rest of the universe evolves at a rate $\underbrace{4.4 \times 10^6}$ times faster than you

$$\left[R_{s_{M87}} = 128 \text{ AU} = 1.92 \times 10^{13} \text{ m} \right]$$

4.4 million times faster