Example 1: Force of Earth on an object m at surface

According to Newton: $F_{E\to m} = \dfrac{G M_E m}{r^2} = \dfrac{G M_E m}{R_E^2}$ (1)

r = Earth-object distance = R_E = radius of Earth

According to Newton's 2nd Law: $F_{E\to m} = m\, a_m$ (2)

Combine eq. (1) & (2): $\cancel{m}\, a_m = F_{E\to m} = \dfrac{G M_E \cancel{m}}{R_E^2}$

m cancels on both sides!!!

$$\Longleftrightarrow \quad \boxed{a_m = \frac{G M_E}{R_E^2}}$$

$M_E = 5.972 \times 10^{24}$ kg

$R_E = 6371 \times 10^{3}$ m

a_m is independent of the mass m

$$= \frac{(6.6743 \times 10^{-11})(5.972 \times 10^{24})}{(6371 \times 10^{3})^2}$$

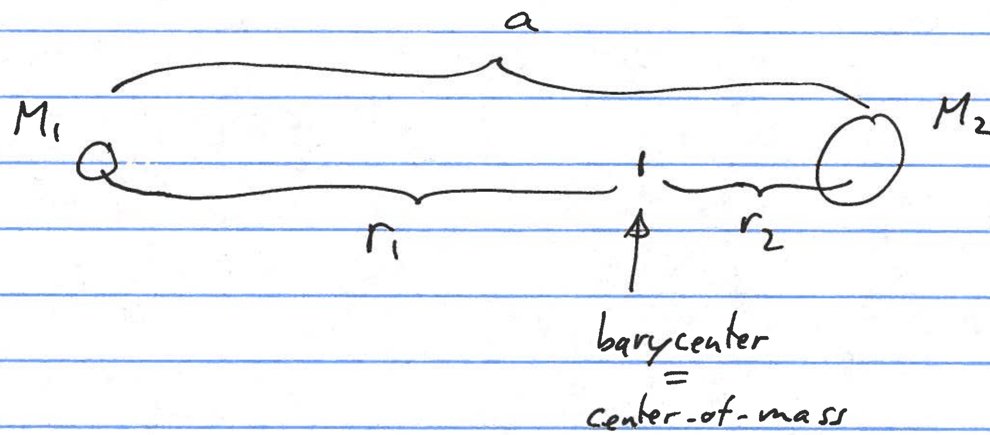
$$\Rightarrow \quad \boxed{a_m = 9.82 \text{ m/s}^2 = g}$$

= acceleration due to gravity on Earth

Student Question

Derivation of Barycenter position formula

Consider the following two mass system:



We know that the barycenter is given by

$$M_1 r_1 = M_2 r_2 \quad (1)$$

By geometry, we know that $r_1 + r_2 = a \quad (2)$

We can rewrite these equations as

$$\begin{cases} r_1 = \dfrac{M_2}{M_1} r_2 & (1) \\ r_1 = a - r_2 & (2) \end{cases}$$

$$(1) = (2) \implies \frac{M_2}{M_1} r_2 = r_1 = a - r_2$$

$$\iff \frac{M_2}{M_1} r_2 + r_2 = a \iff r_2\left(\frac{M_2}{M_1} + 1\right) = a$$

$$\iff r_2\left(\frac{M_2 + M_1}{M_1}\right) = a \iff \boxed{r_2 = a\,\frac{M_1}{M_2 + M_1}}$$

(see lecture 7 slide 12 video A) → distance from barycenter to M_2