Friday, September 4, 2020

Example 1: Newton's version of Kepler's 3rd law for determining planetary mass

If we assume that $M_{Moon} \ll M_{Earth}$, then we can measure M_{Earth} with the Moon's orbit

Moon's orbital parameters:

$$\begin{cases} a = 384\,399 \text{ km} \quad \text{(average Earth–Moon distance)} \\ \quad \simeq 3.84 \times 10^8 \text{ m} \\ T = 27.3 \text{ days} \quad \text{(Moon's orbital period)} \\ \quad = 27.3 \times 24 \times 60 \times 60 = 2358720 \\ \quad \simeq 2.36 \times 10^6 \text{ s} \end{cases}$$

Newton's version of Kepler's 3rd Law:

$$T^2 = \frac{4\pi^2}{G(\underbrace{M_{Earth} + M_{Moon}}_{\simeq M_{Earth}})} a^3 \Rightarrow T^2 \simeq \frac{4\pi^2}{G\, M_{Earth}} a^3$$

$$\Rightarrow M_{Earth} \simeq \frac{4\pi^2}{G} \frac{a^3}{T^2} = \frac{4(3.1415926)^2}{(6.6743 \times 10^{-11})} \frac{(3.84 \times 10^8)^3}{(2.36 \times 10^6)^2}$$

G: $6.6743 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$

$$\simeq 6.01 \times 10^{24} \text{ kg}$$

$$\Rightarrow \boxed{M_{Earth} \simeq 6.01 \times 10^{24} \text{ kg}}$$

Real mass of Earth: $M_{Earth} = 5.97 \times 10^{24}$ kg