Today's Topics

Wednesday, September 2, 2020 (Week 2, lecture 7) – Chapter 3, 4.6.

A. Center of Mass

B. Angular momentum

C. Escape velocity

D. Tides

Center of Mass

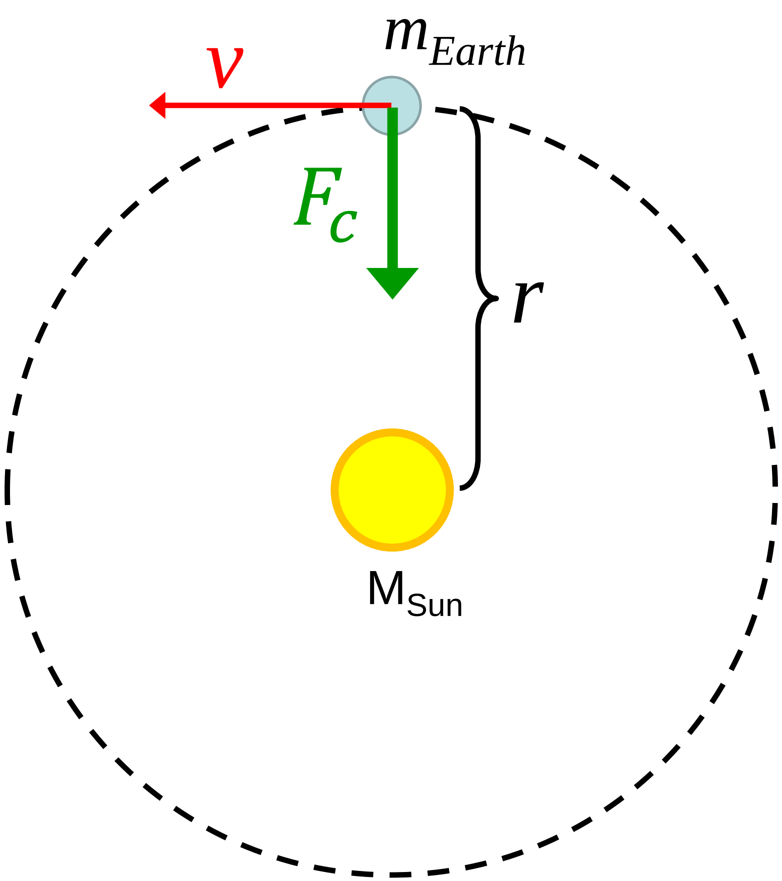
Circular Motion Example: Earth's orbit of Sun

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

Force of gravity on Earth from Sun:

$$F_{gravity,\, S\rightarrow E} = G\frac{m_{Earth} M_{Sun}}{r^2}$$



Circular Motion Example: Earth's orbit of Sun

Centripetal force needed to keep Earth on a circular orbit:

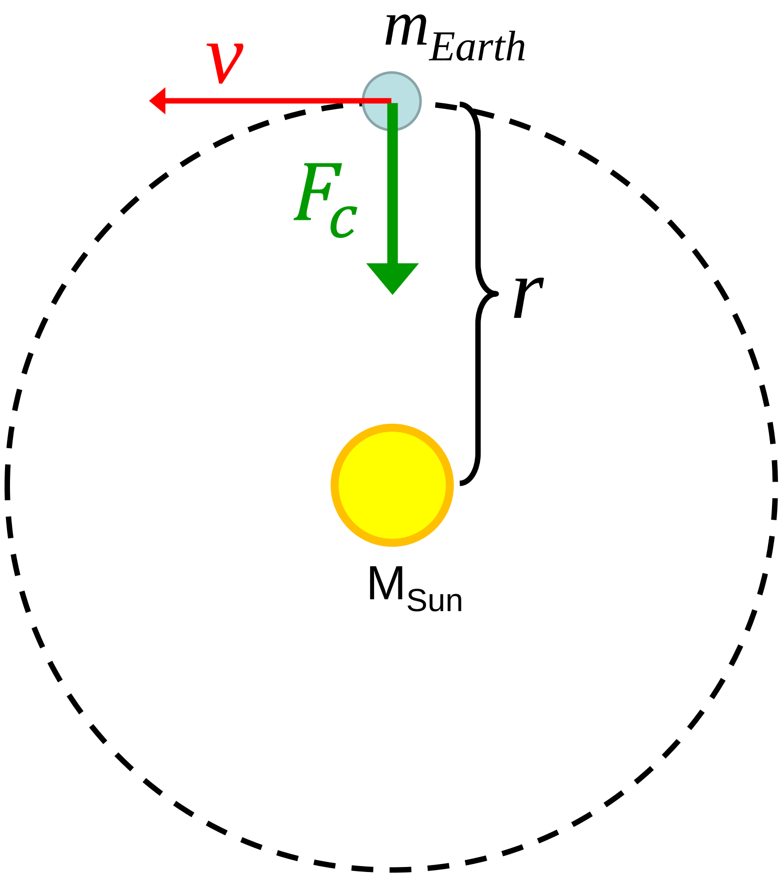
$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

Force of gravity on Earth from Sun:

$$F_{gravity,\, S\rightarrow E} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

Force of gravity on Sun from Earth is the same:

$$F_{gravity,\, E\rightarrow S} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

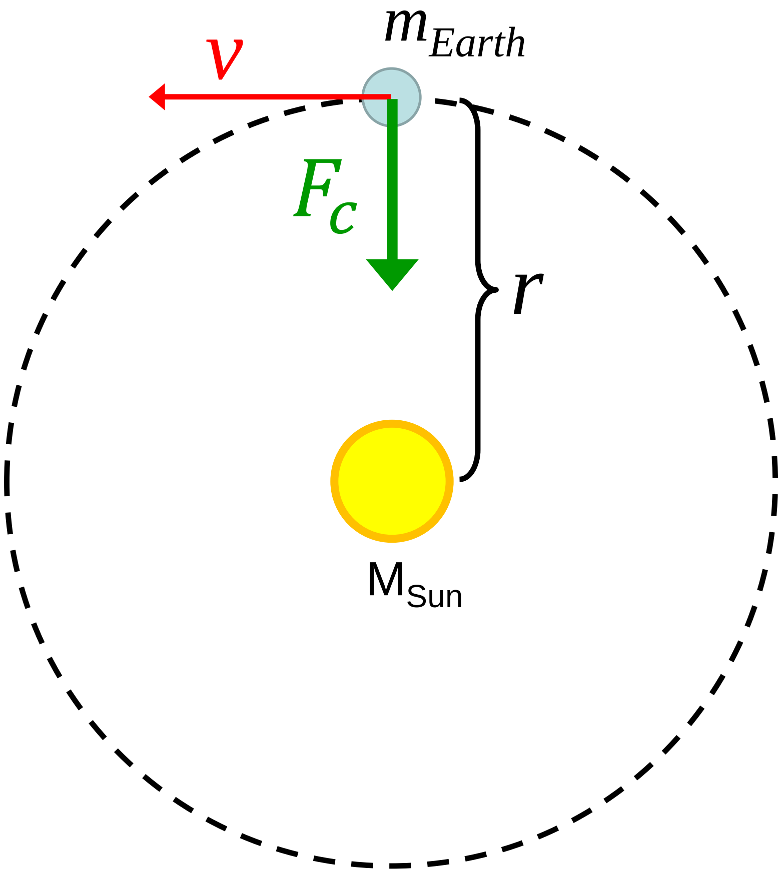


The Sun is really heavy compared to Earth, so it's hard to move it

Circular Motion Example: Earth's orbit of Sun

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$



Force of gravity on Earth from Sun:

$$F_{gravity,\, S\to E} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

Force of gravity on Sun from Earth is the same:

$$F_{gravity,\, E\to S} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

The Sun is really heavy compared to Earth, so it's hard to move it

WHAT IF: What would happen if the Earth and Sun were comparable in mass ?

Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

This formula is in SI units

T = orbital period in seconds

a = semimajor axis in meters

$M_{1,2}$ =Mass of orbiting objects in Kg

G = 6.6743×10^{-11} $m^3/Kg.s^2$

Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

This formula is in SI units

T = orbital period in seconds

a = semimajor axis in meters

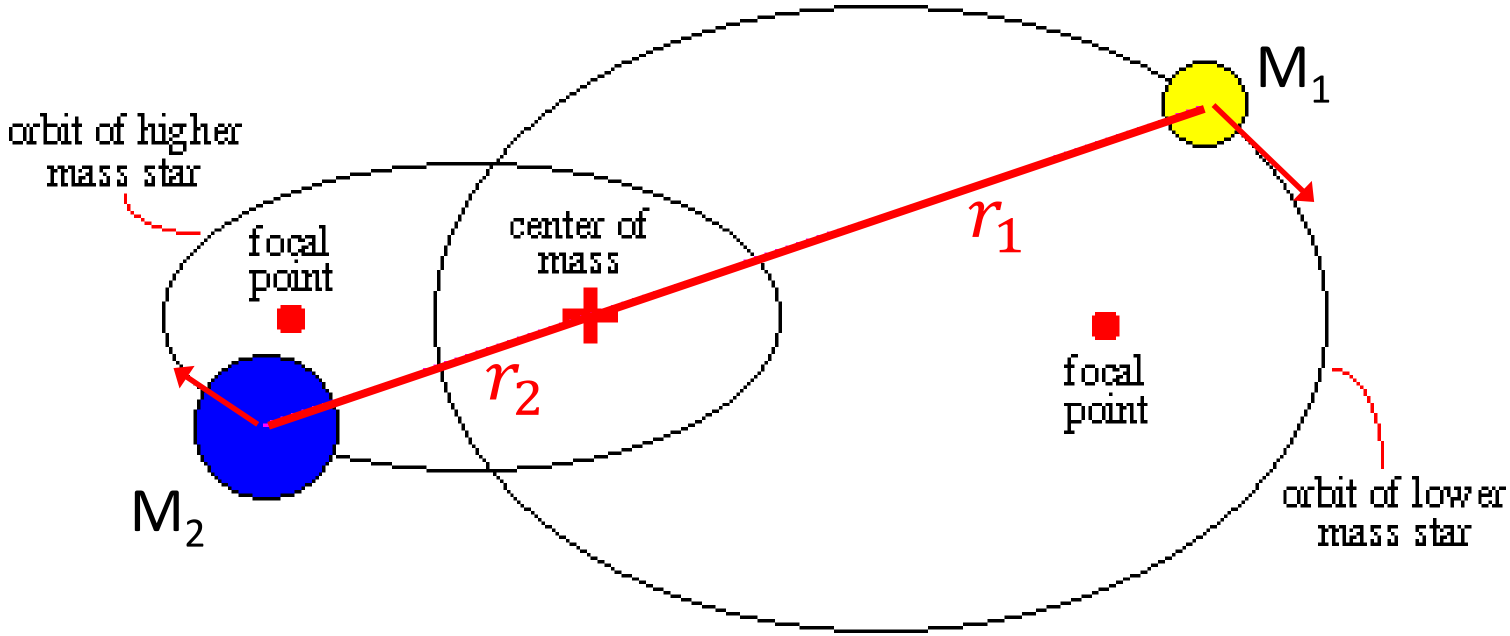
$M_{1,2}$ =Mass of orbiting objects in Kg

G = 6.6743×10^{-11} $m^3/Kg.s^2$

WHAT IF: What happens to the orbits if the M_1 and M_2 are comparable ?

What happens when $M_1 \simeq M_2$?

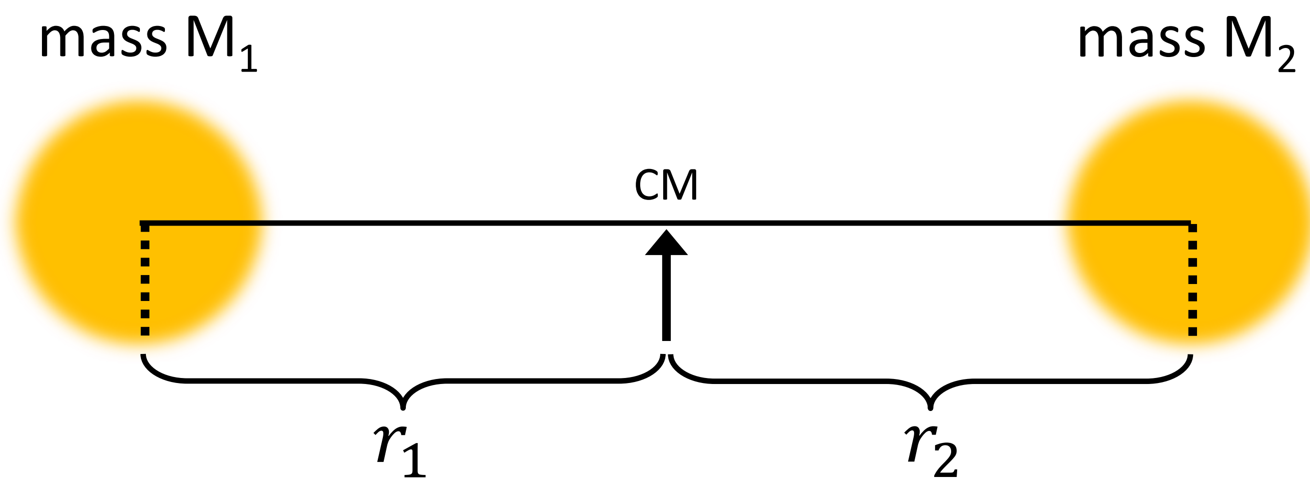
The **center of mass** of M_1 and M_2 serves as the orbiting ellipse focus.



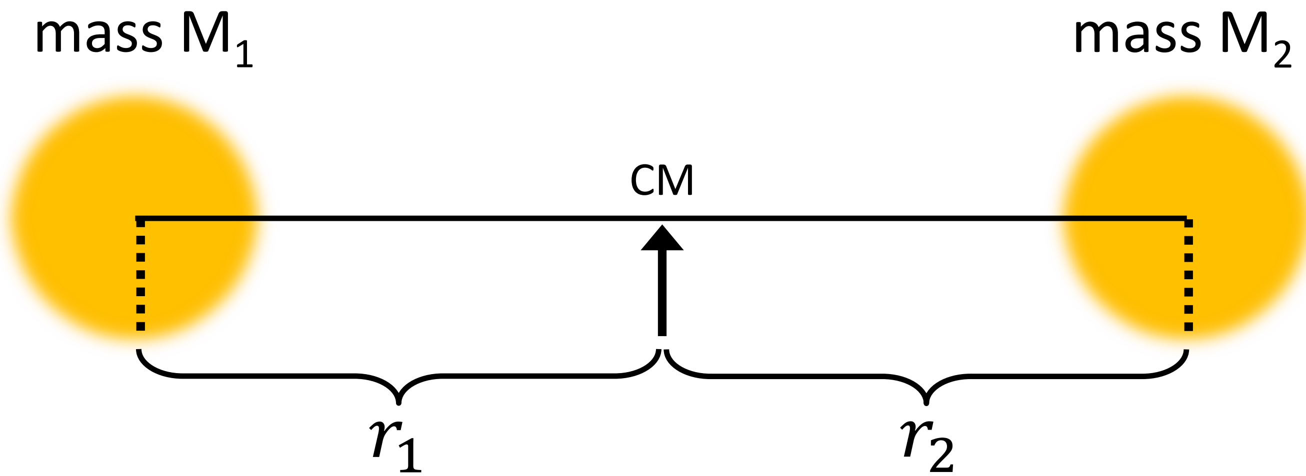
[adapted from http://abyss.uoregon.edu]

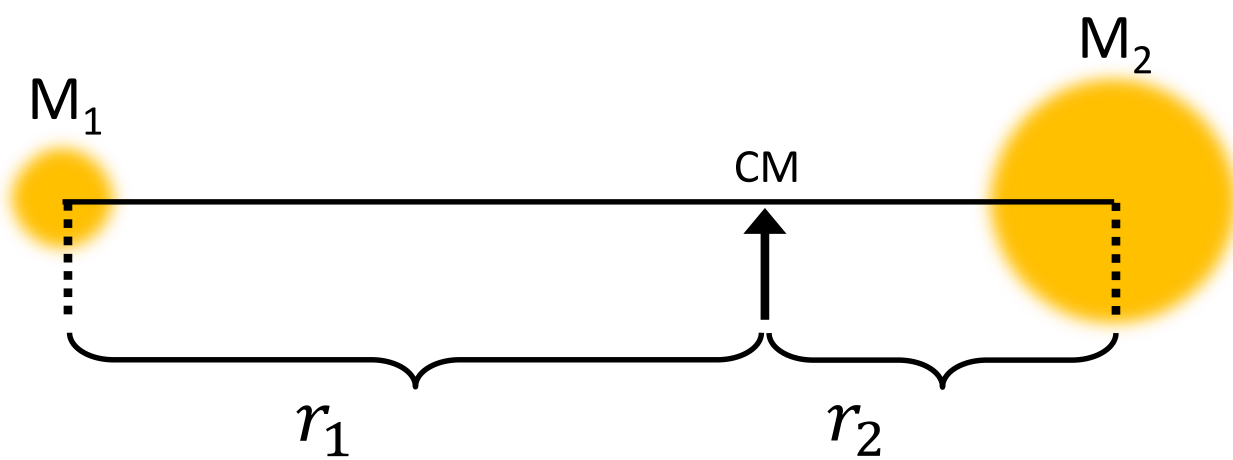
Semimajor axis "a":
The coordinate "$r = r_1 + r_2$" is the distance between the two masses. It also describes an ellipse (not shown), whose semimajor axis "a" is used in Newton's version of Kepler's 3rd law.

Center of Mass

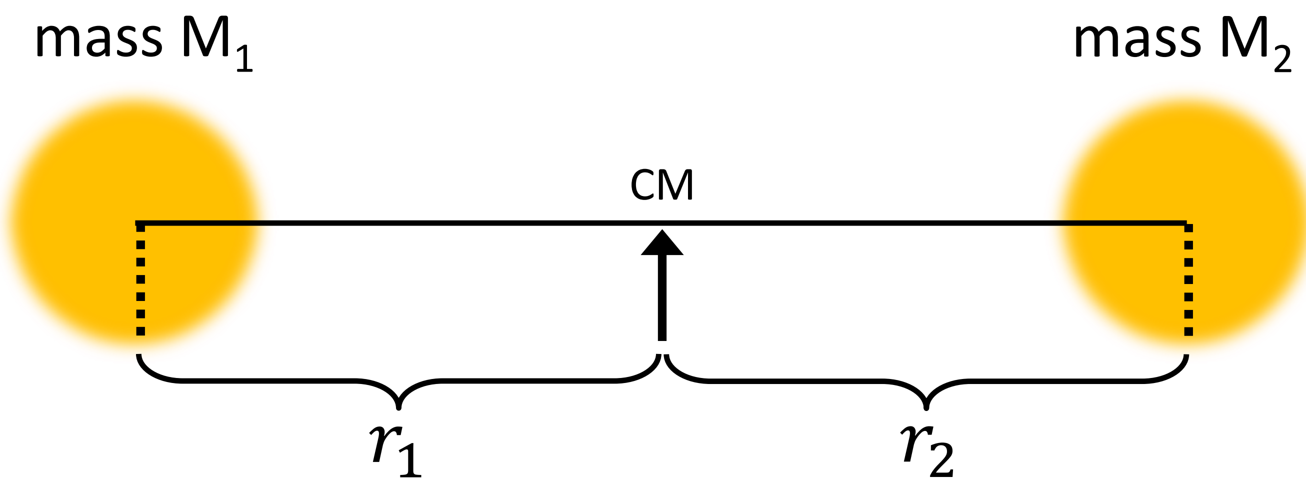


Center of Mass

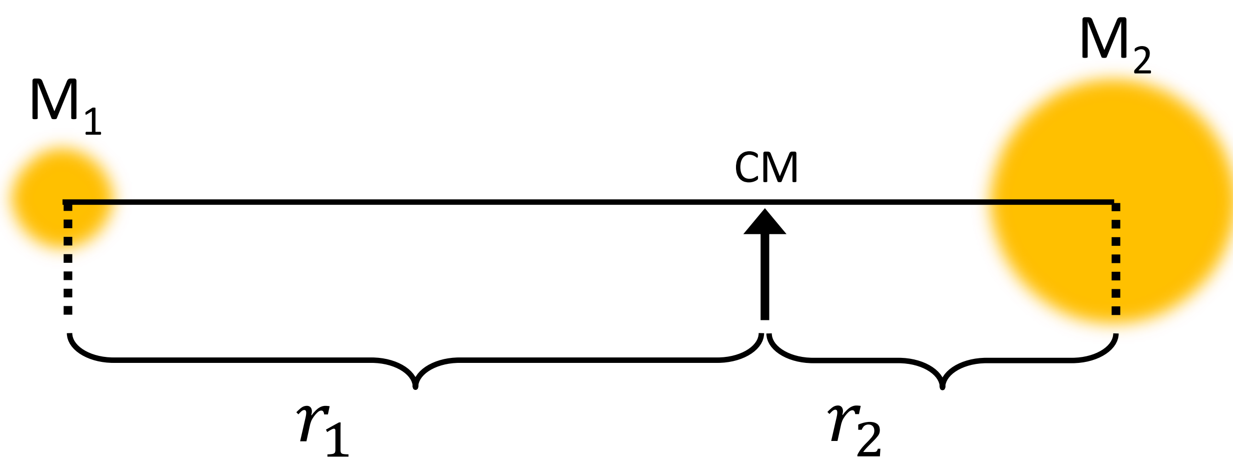




Center of Mass



Center of mass is located such that $\boxed{M_1 r_1 = M_2 r_2}$
(or “barycenter”)



Some Barycenters

M_2 - M_1: $r_2 = a\frac{M_1}{M_1+M_2}$ = distance from CM to M_2

Sun-Earth: $r_2 = 448\ km = 3.0 \times 10^{-6}$ AU

Earth-Moon: $r_2 = 4{,}670$ km with $a = 384{,}000$ km

= 73% of Earth's radius

Some Barycenters

M_2 - M_1: $r_2 = a\frac{M_1}{M_1+M_2}$ = distance from CM to M_2

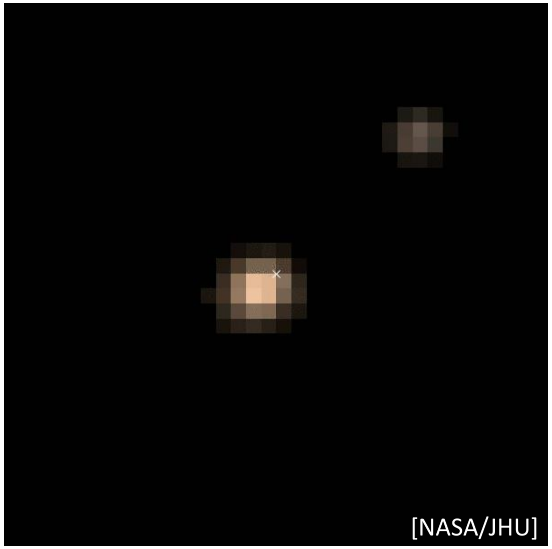
Sun-Earth: $r_2 = 448\ km = 3.0 \times 10^{-6}$ AU

Earth-Moon: $r_2 = 4{,}670$ km with $a = 384{,}000$ km

= 73% of Earth's radius

Pluto – Charon:

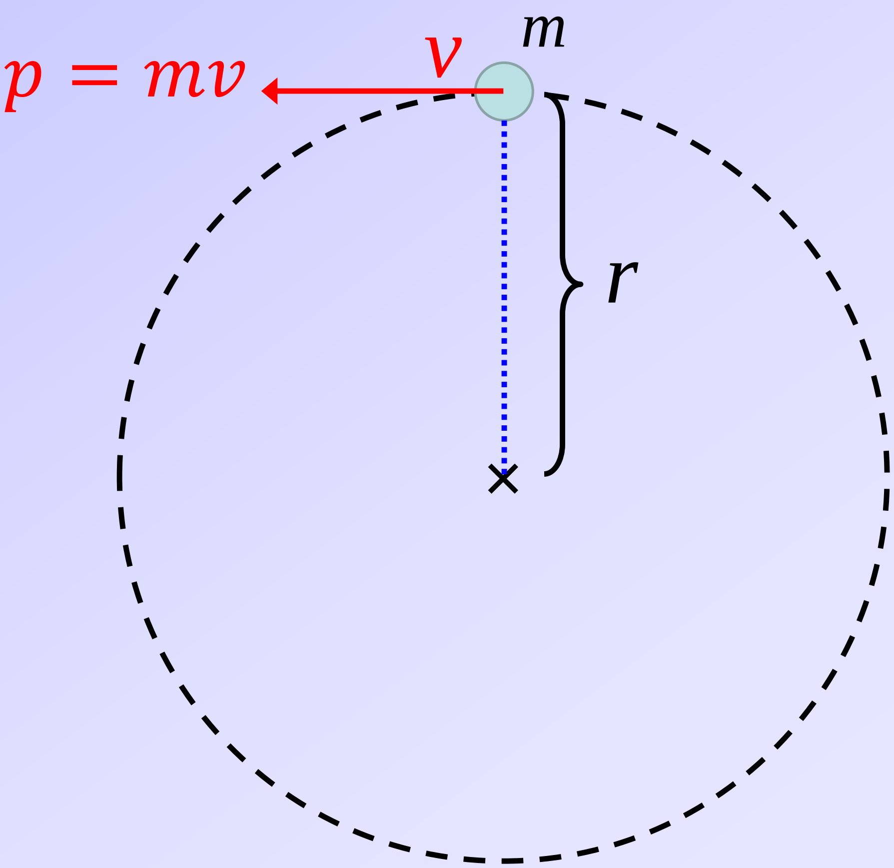
orbital period T = 6.4 days



Conservation of Angular Momentum (1)

angular momentum $= L =$ momentum $\times$ radius

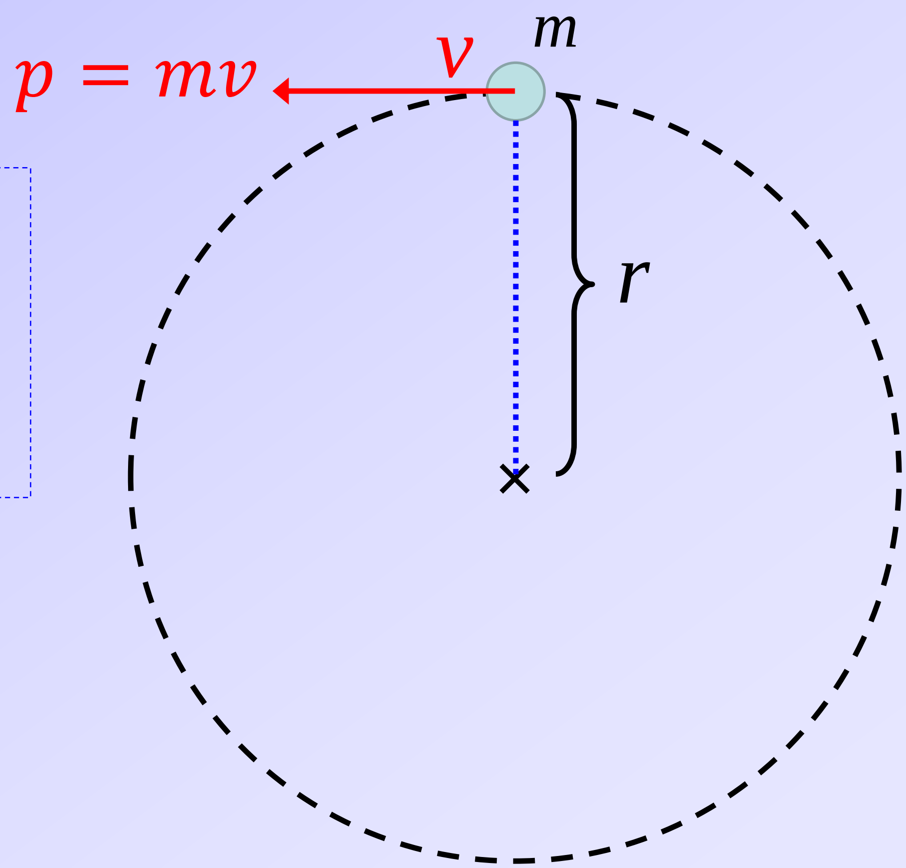
$= p \times r$ ... $= mvr$ for circular motion



Conservation of Angular Momentum (1)

angular momentum $= L =$ momentum $\times$ radius

$= p \times r$ ... $= mvr$ for circular motion



total angular momentum

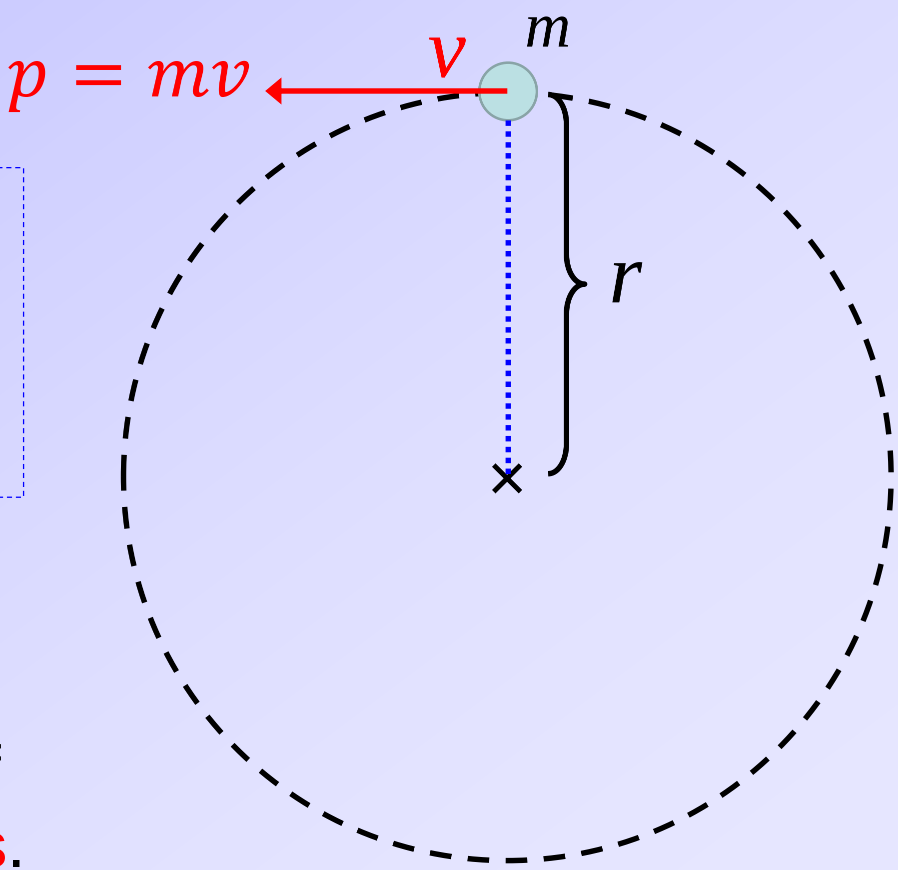
=

sum of the angular momenta of all the sub-parts of a system

Conservation of Angular Momentum (1)

angular momentum $= L =$ momentum $\times$ radius

$= p \times r$ **...** $= mvr$ for circular motion



total angular momentum

=

sum of the angular momenta of all the sub-parts of a system

Conservation Law

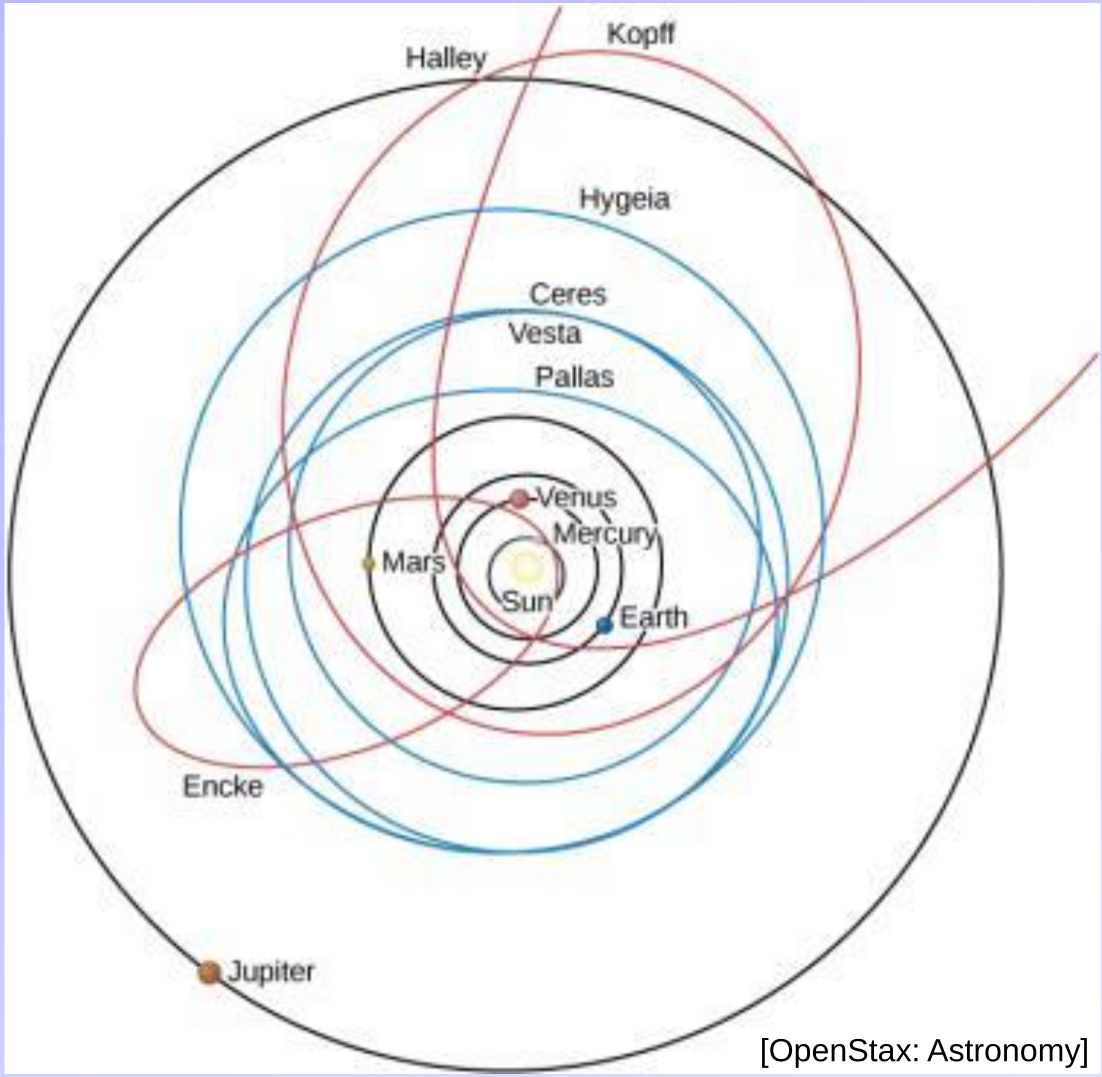
The total angular momentum of a closed system never changes.

Conservation of Angular Momentum (2)

[OpenStax: Astronomy]

- When a spinning figure skater **brings in her arms**, their distance from her spin center is **smaller**, so her speed increases.
- When her **arms are out**, their distance from the spin center is **greater**, so she **slows down**.

Conservation of Angular Momentum



[OpenStax: Astronomy]

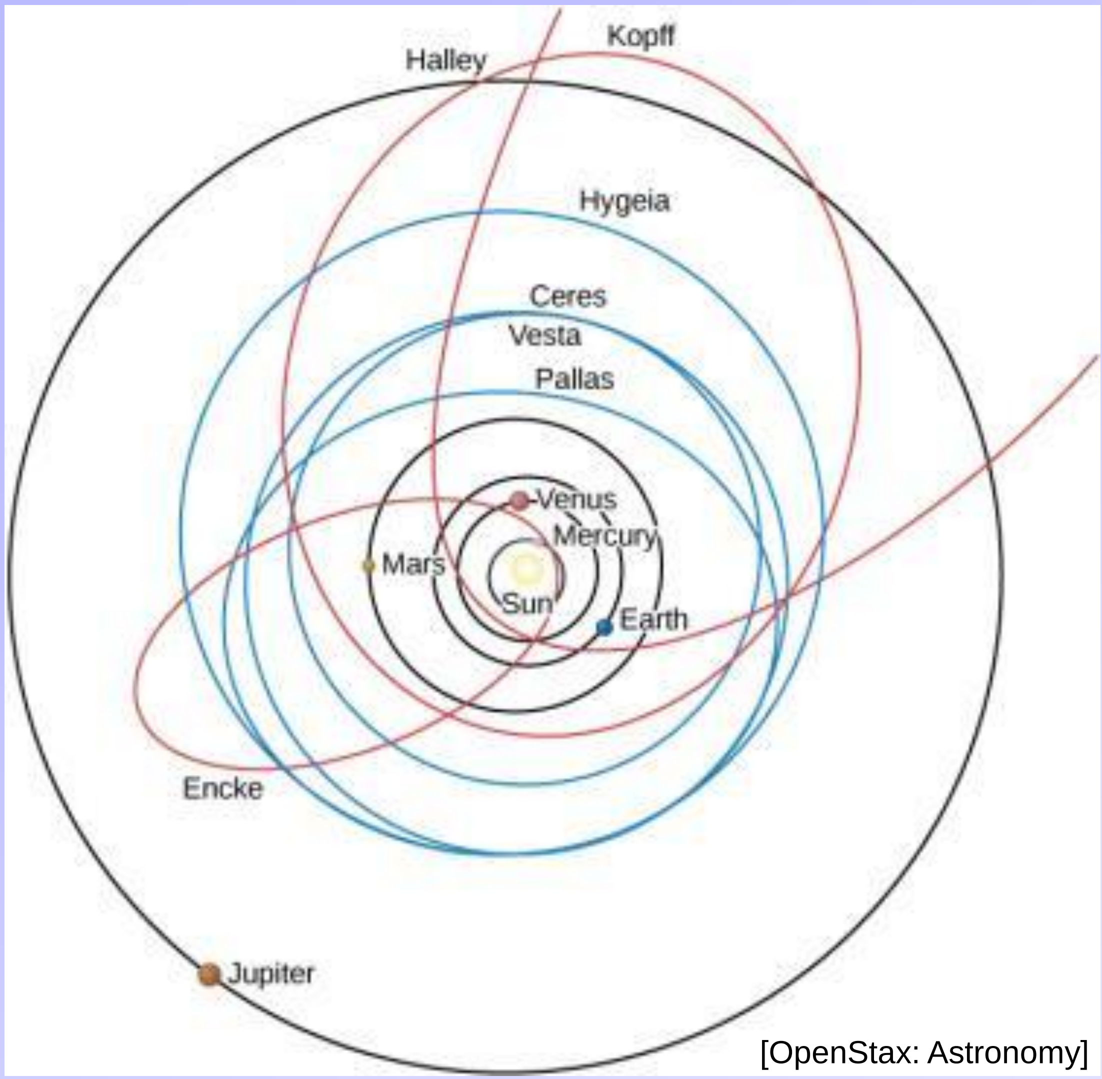
Planets (black), asteroids (blue), comets (red)

The multiple planets, asteroids, and comets all interact and modify each others orbits.

→ **Individual angular momenta change.**

→ **Total angular momentum of Solar System is constant.**

Conservation of Angular Momentum



[OpenStax: Astronomy]

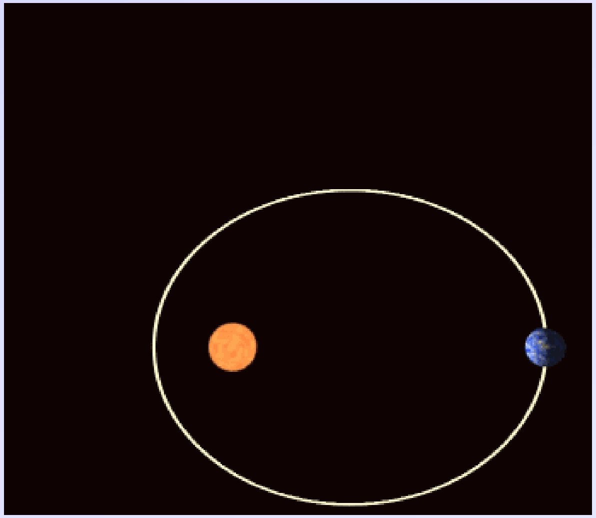
Planets (black), asteroids (blue), comets (red)

The multiple planets, asteroids, and comets all interact and modify each others orbits.

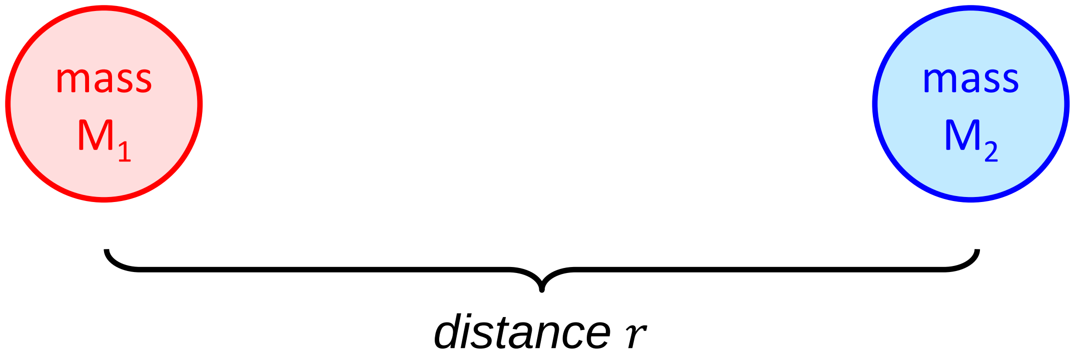
→ **Individual angular momenta change.**

→ **Total angular momentum of Solar System is constant.**

Example: Apsidal Precession



Gravitational Potential Energy



Stored gravitational energy $= E_{potential} = -G\,\frac{M_1 M_2}{r}$

Total Energy $= E_{total} = E_{potential} + E_{kinetic}$

For 2 orbiting bodies (e.g. Sun + Earth): $E_{total} < 0$

For 2 unbound bodies (Earth + Mars rocket): $E_{total} > 0$

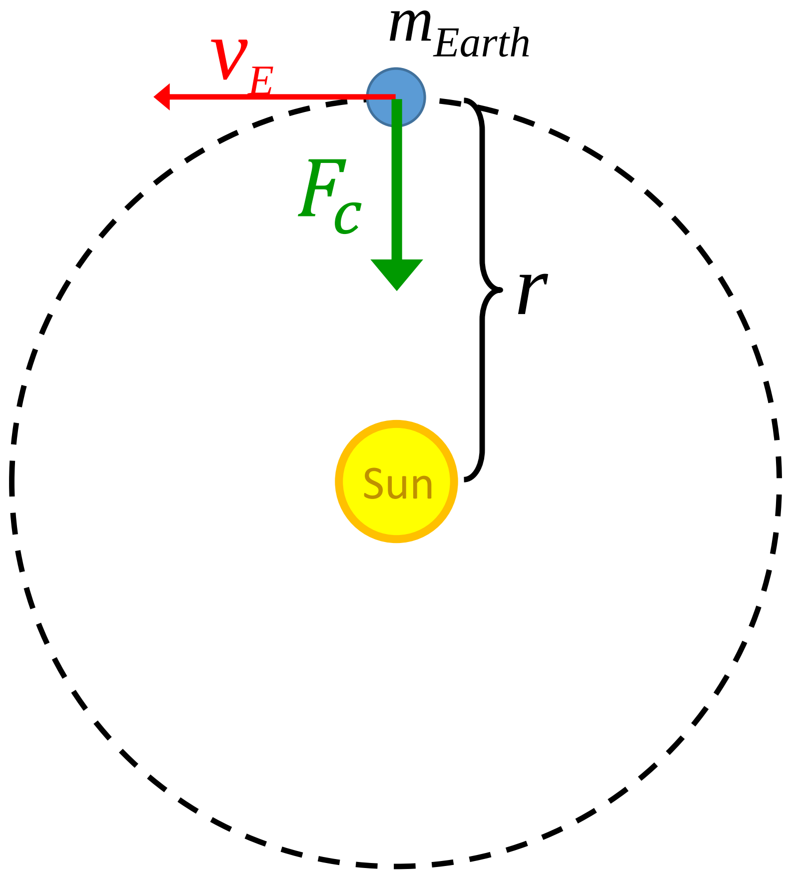
Bound Orbital Energy

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

Force of gravity on Earth from Sun:

$$F_{gravity,\, S\rightarrow E} = G\frac{m_{Earth} M_{Sun}}{r^2}$$



Bound Orbital Energy

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

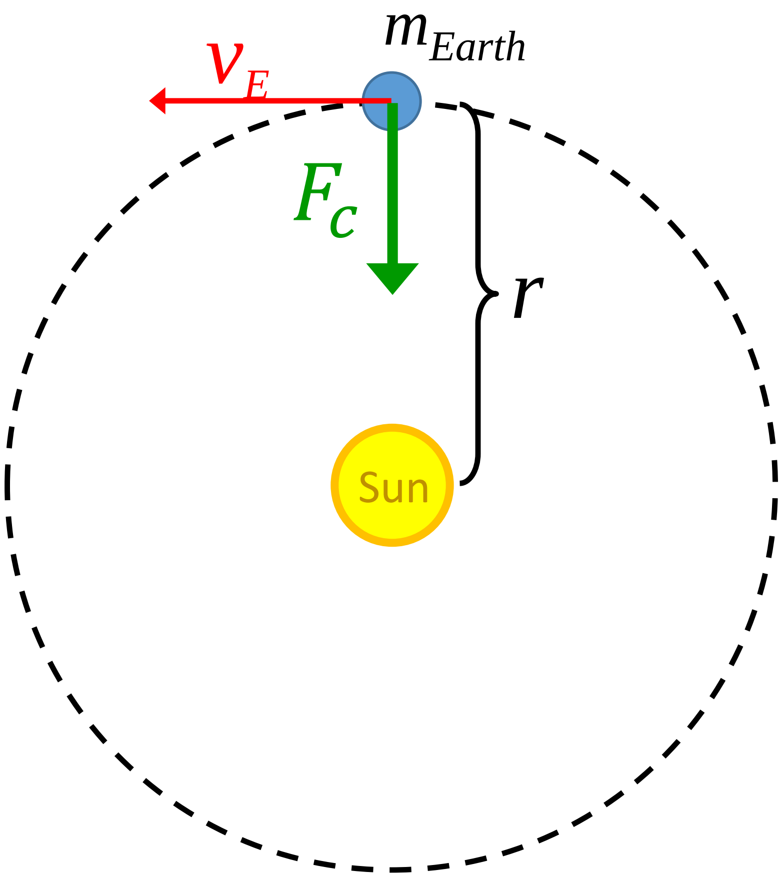
Force of gravity on Earth from Sun:

$$F_{gravity,\, S\to E} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

The **centripetal force** that pulls on Earth to make it orbit the Sun **is gravity:**

$$F_c = F_{gravity,\, S\to E}$$

$$\Leftrightarrow \quad \frac{m_{Earth} v_{Earth}^2}{r} = G\frac{m_{Earth} M_{Sun}}{r^2}$$



Bound Orbital Energy

Centripetal force needed to keep
Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

Force of gravity on Earth from Sun:

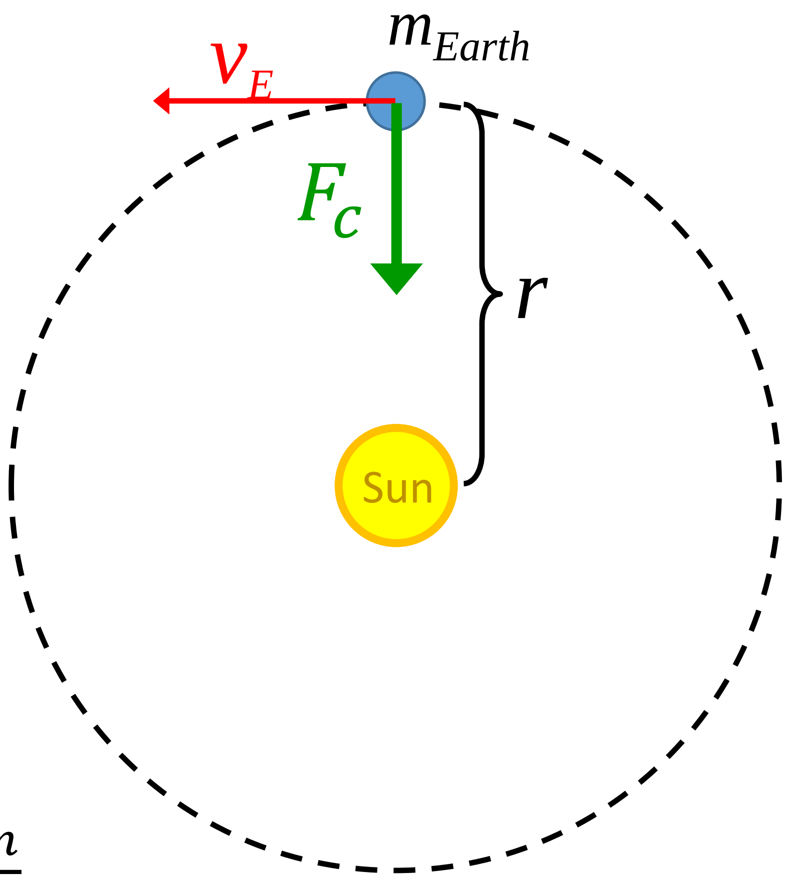
$$F_{gravity,\, S \to E} = G \frac{m_{Earth} M_{Sun}}{r^2}$$

The **centripetal force** that pulls on Earth to make it orbit the Sun **is gravity:**

$$F_c = F_{gravity,\, S \to E}$$

$$\Leftrightarrow \quad \frac{m_{Earth} v_{Earth}^2}{r} = G \frac{m_{Earth} M_{Sun}}{r^2}$$

$$\Leftrightarrow \quad \frac{1}{2} m_{Earth} v_{Earth}^2 = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$



Bound Orbital Energy

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$

Force of gravity on Earth from Sun:

$$F_{gravity,\, S\rightarrow E} = G\,\frac{m_{Earth} M_{Sun}}{r^2}$$

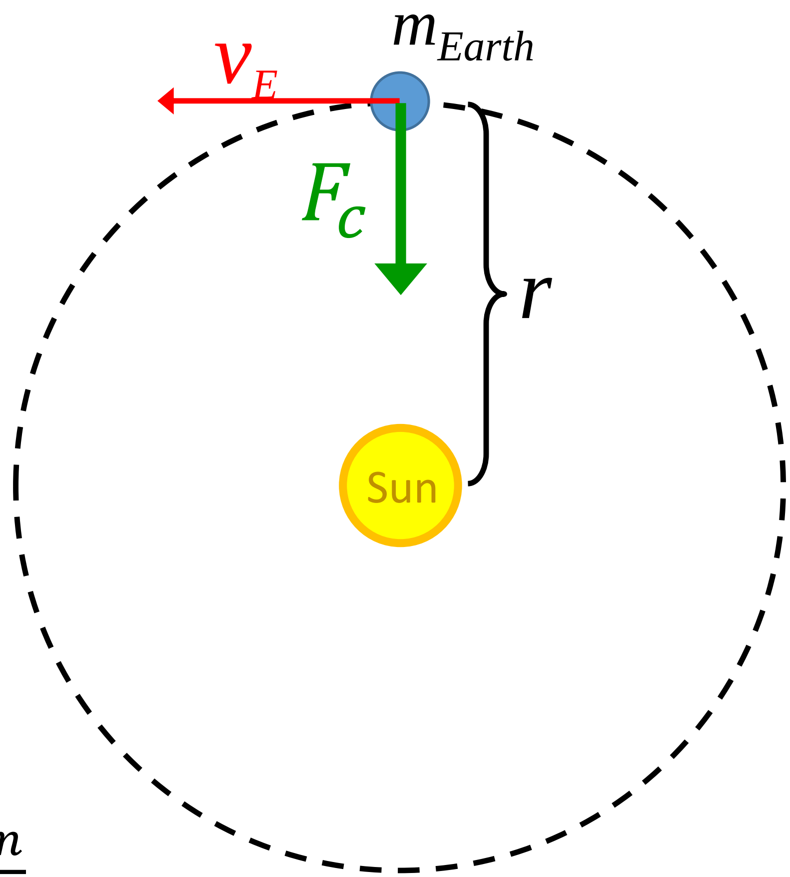
The **centripetal force** that pulls on Earth to make it orbit the Sun **is gravity:**

$$F_c = F_{gravity,\, S\rightarrow E}$$

$$\Leftrightarrow \quad \frac{m_{Earth} v_{Earth}^2}{r} = G\,\frac{m_{Earth} M_{Sun}}{r^2}$$

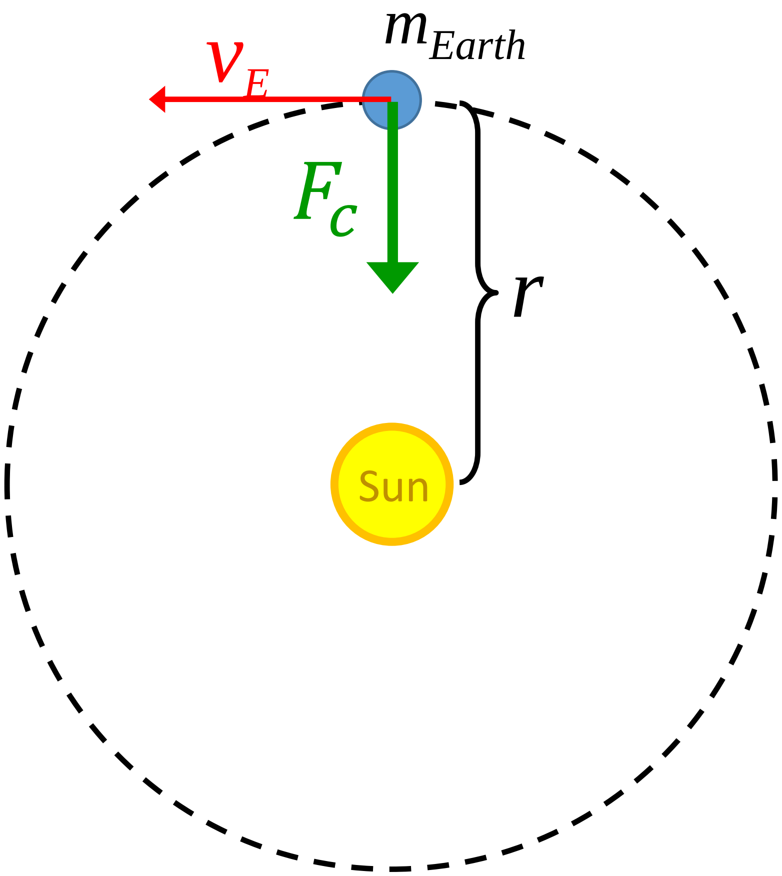
$$\Leftrightarrow \quad \frac{1}{2} m_{Earth} v_{Earth}^2 = \frac{1}{2} G\,\frac{m_{Earth} M_{Sun}}{r}$$

$$\Leftrightarrow \quad E_{kinetic} = \frac{1}{2} m_{Earth} v_{Earth}^2 = \frac{1}{2} G\,\frac{m_{Earth} M_{Sun}}{r}$$



Bound Orbital Energy

$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

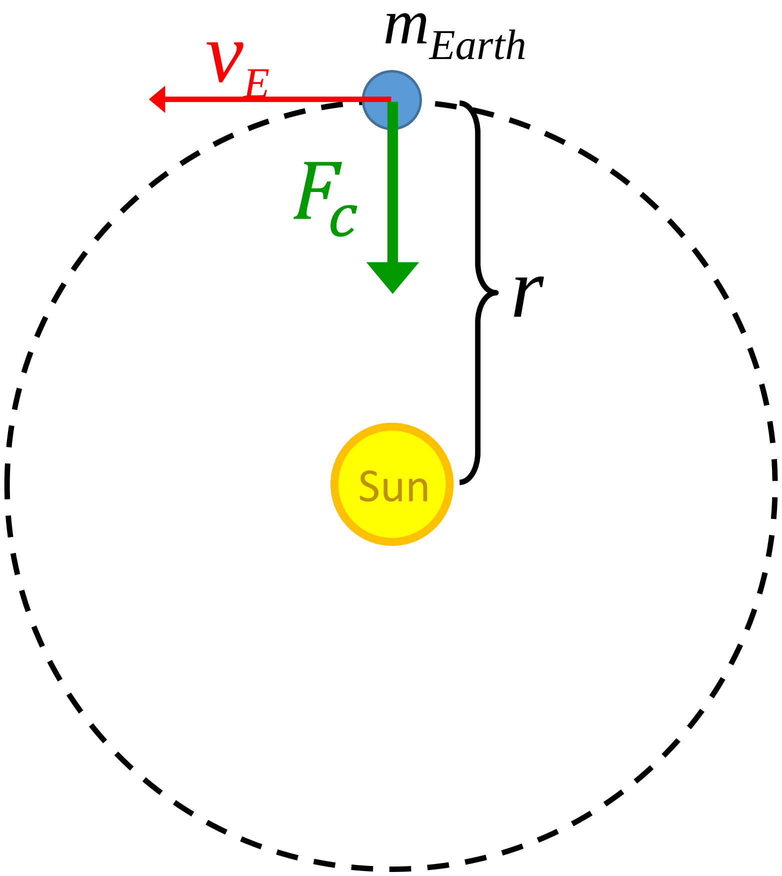


Bound Orbital Energy

$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

Total Energy:

$$E_{Total} = E_{kinetic} + E_{potential}$$



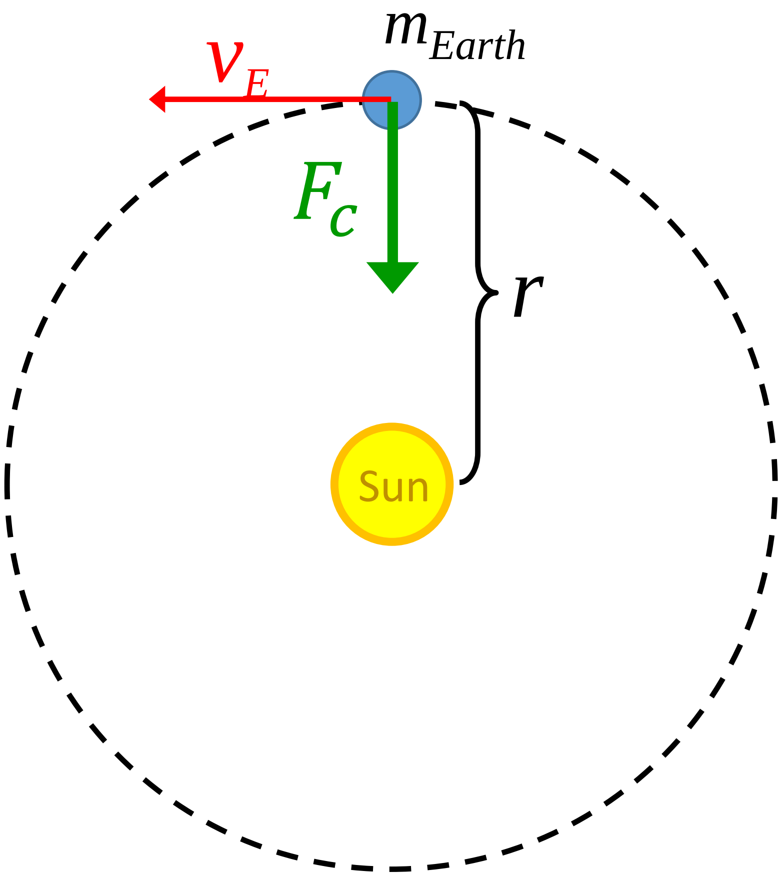
Bound Orbital Energy

$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

Total Energy:

$$E_{Total} = E_{kinetic} + E_{potential}$$

$$= \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r} - G \frac{m_{Earth} M_{Sun}}{r}$$



Bound Orbital Energy

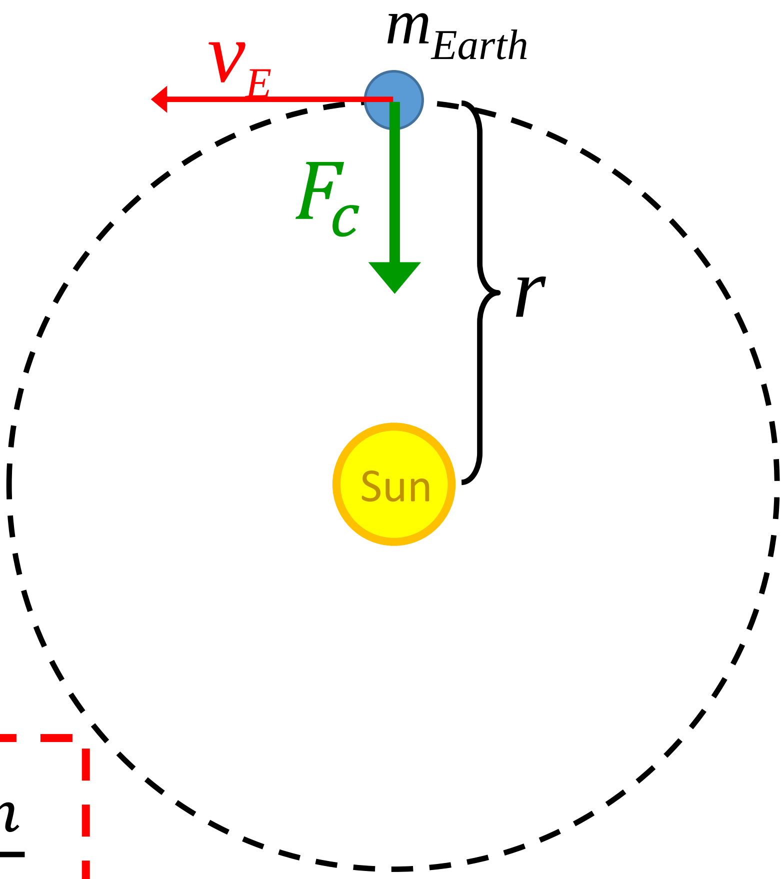
$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

Total Energy:

$$E_{Total} = E_{kinetic} + E_{potential}$$

$$= \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r} - G \frac{m_{Earth} M_{Sun}}{r}$$

$$\Leftrightarrow \boxed{E_{Total} = -\frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}}$$



Bound Orbital Energy

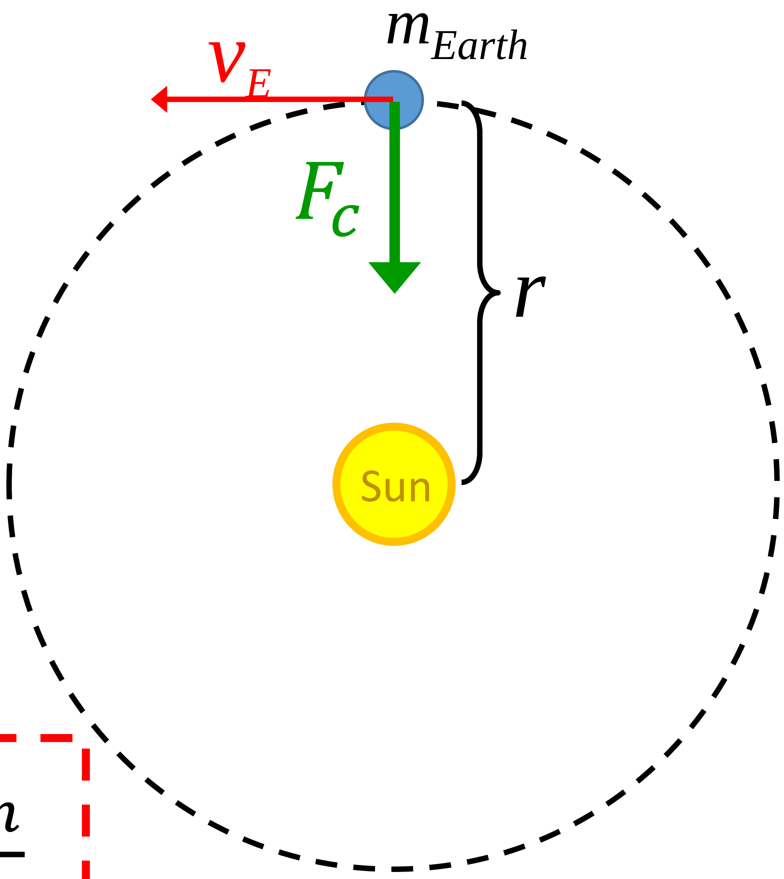
$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

Total Energy:

$$E_{Total} = E_{kinetic} + E_{potential}$$

$$= \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r} - G \frac{m_{Earth} M_{Sun}}{r}$$

$$\Leftrightarrow \boxed{E_{Total} = -\frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}}$$



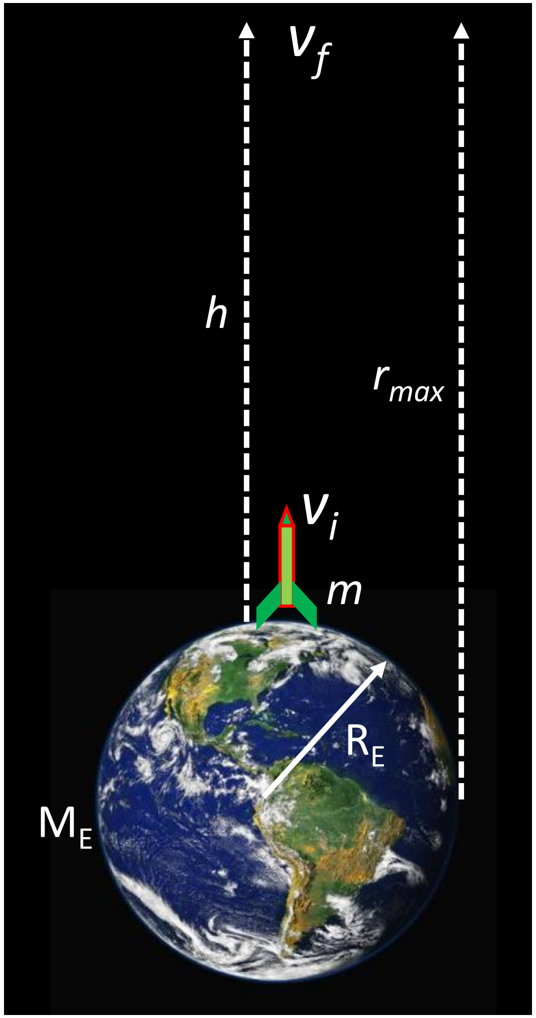
The bound orbital energy is negative: $E_{Total} < 0$

Example: *When a rocket wants to orbit another planet it has to slow (lower its energy) in order to go into orbit.*

Escape Velocity

Question

What is the minimum velocity needed to escape Earth's gravity?



$$\boxed{v_{escape} = \sqrt{\frac{2GM_E}{R_E}}}$$

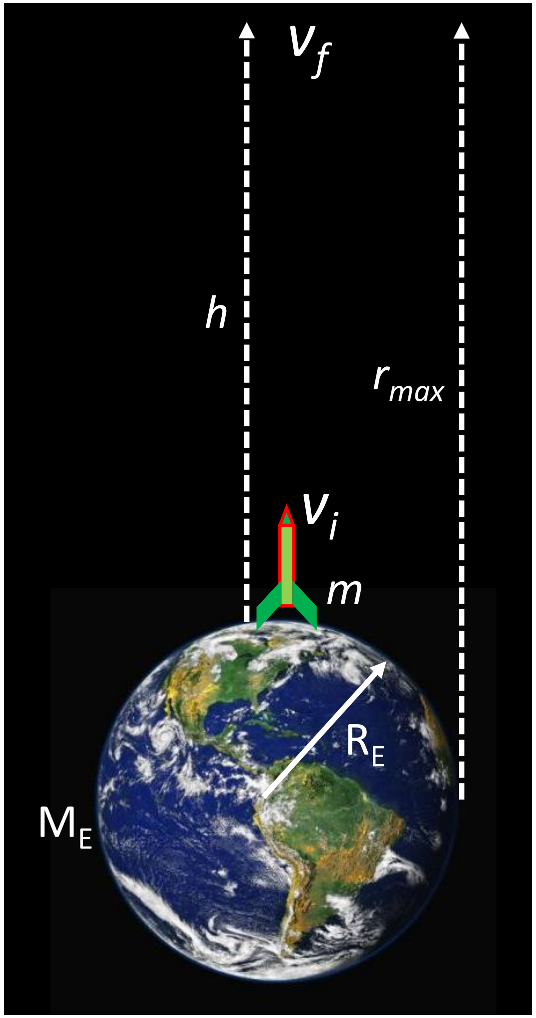
= 11.2 km/s on Earth

Note 1: escape velocity depends on your starting point.

Note 2: Since the Earth spins, objects at "rest" close to the equator already have a significant velocity.

→ Rockets are typically launched close to the equator (or in Florida)

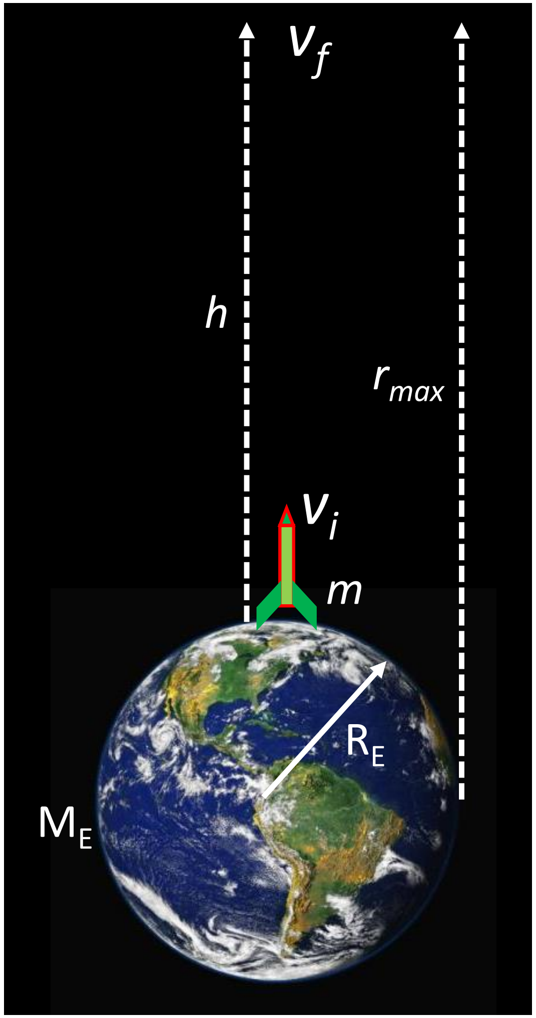
Escape Velocity: Derivation



The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

Escape Velocity: Derivation



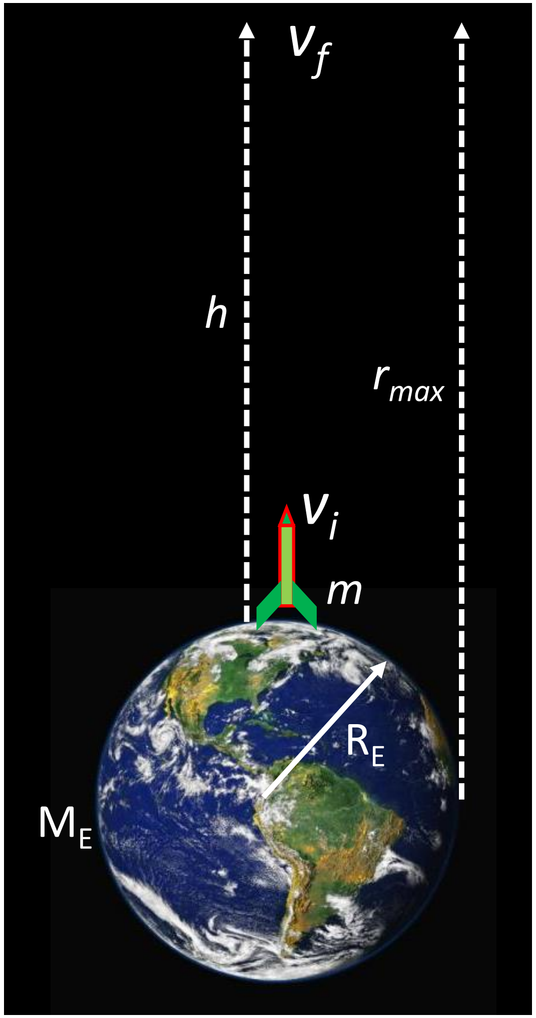
The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

Conservation of total energy:

$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E}}_{\text{at launch}}$$

Escape Velocity: Derivation



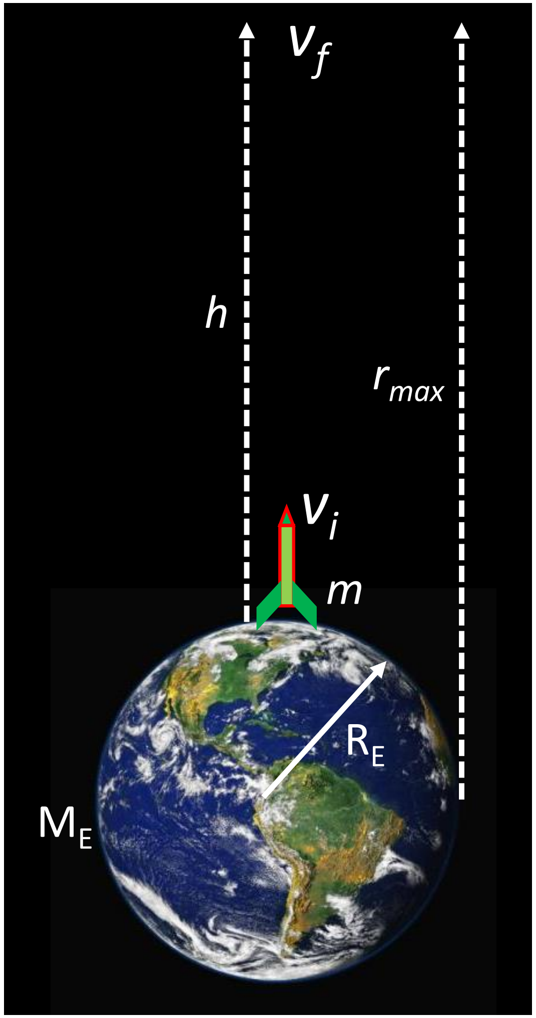
The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

Conservation of total energy:

$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E}}_{\text{at launch}} = \underbrace{-\frac{GM_Em}{r_{max}}}_{\text{at max height}}$$

Escape Velocity: Derivation



The projectile reaches its maximum altitude when

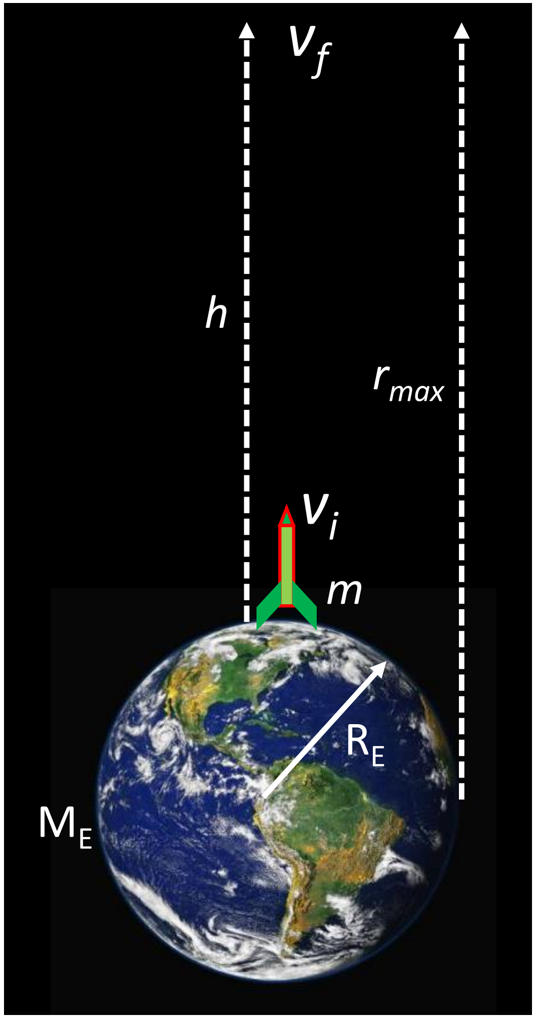
$$v_{final} = v_f = 0$$

Conservation of total energy:

$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E}}_{\text{at launch}} = \underbrace{-\frac{GM_E m}{r_{max}}}_{\text{at max height}}$$

Solve for v_i: $$\frac{1}{2}mv_i^2 = GM_E m\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$$

Escape Velocity: Derivation



The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

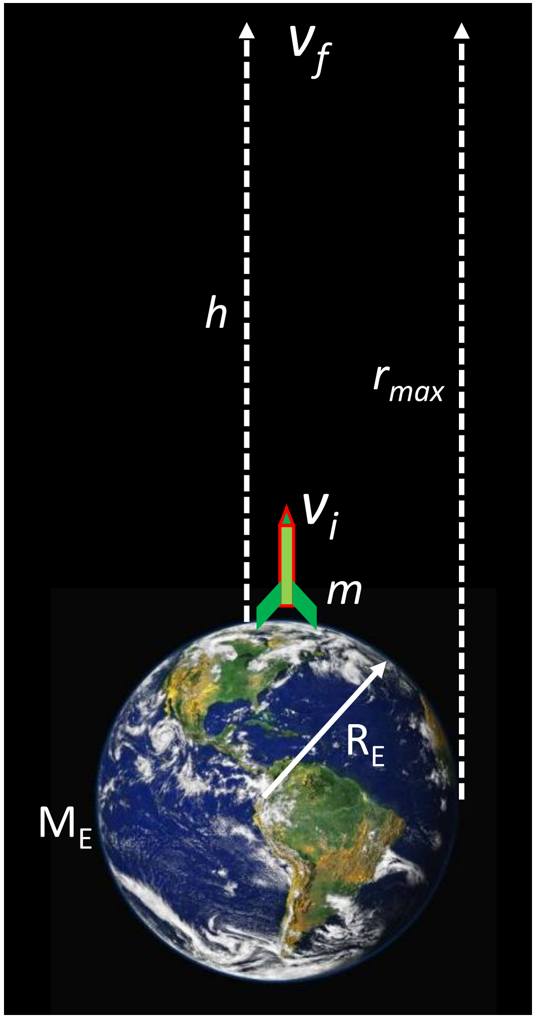
Conservation of total energy:

$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E}}_{\text{at launch}} = \underbrace{-\frac{GM_Em}{r_{max}}}_{\text{at max height}}$$

Solve for v_i: $\frac{1}{2}mv_i^2 = GM_Em\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$

The projectile just barely escapes Earth's gravity when $v_{final} = 0$ at $r_{max} \to \infty$:

Escape Velocity: Derivation



The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

Conservation of total energy:

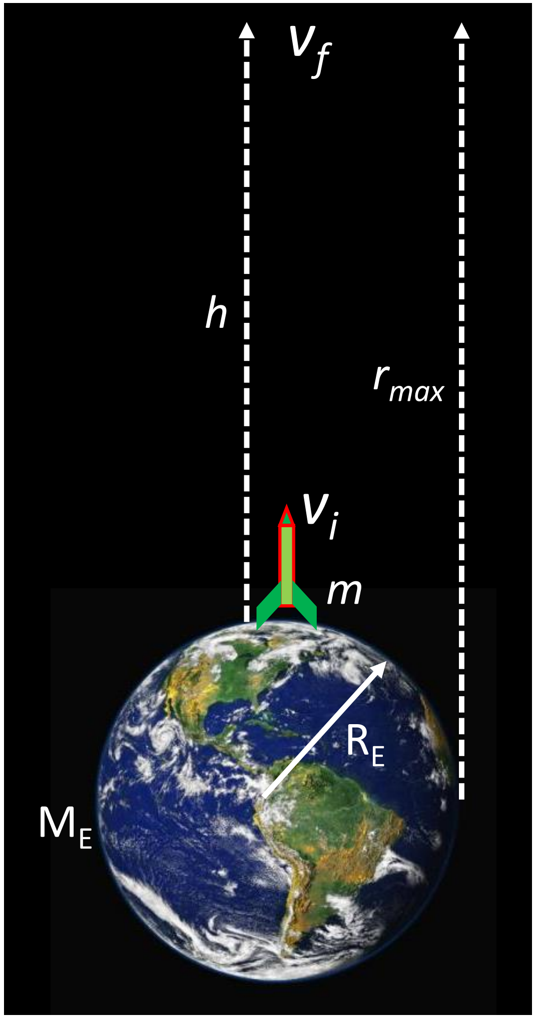
$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E}}_{\text{at launch}} = \underbrace{-\frac{GM_Em}{r_{max}}}_{\text{at max height}}$$

Solve for v_i: $$\frac{1}{2}mv_i^2 = GM_Em\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$$

The projectile just barely escapes Earth's gravity when $v_{final} = 0$ at $r_{max} \to \infty$:

$$v_{escape}^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max} \to \infty}\right)$$

Escape Velocity: Derivation



The projectile reaches its maximum altitude when

$$v_{final} = v_f = 0$$

Conservation of total energy:

$$E_{total} = \underbrace{\frac{1}{2}mv_i^2 - \frac{GM_Em}{R_E}}_{\text{at launch}} = \underbrace{-\frac{GM_Em}{r_{max}}}_{\text{at max height}}$$

Solve for v_i: $$\frac{1}{2}mv_i^2 = GM_Em\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$$

The projectile just barely escapes Earth's gravity when $v_{final} = 0$ at $r_{max} \to \infty$:

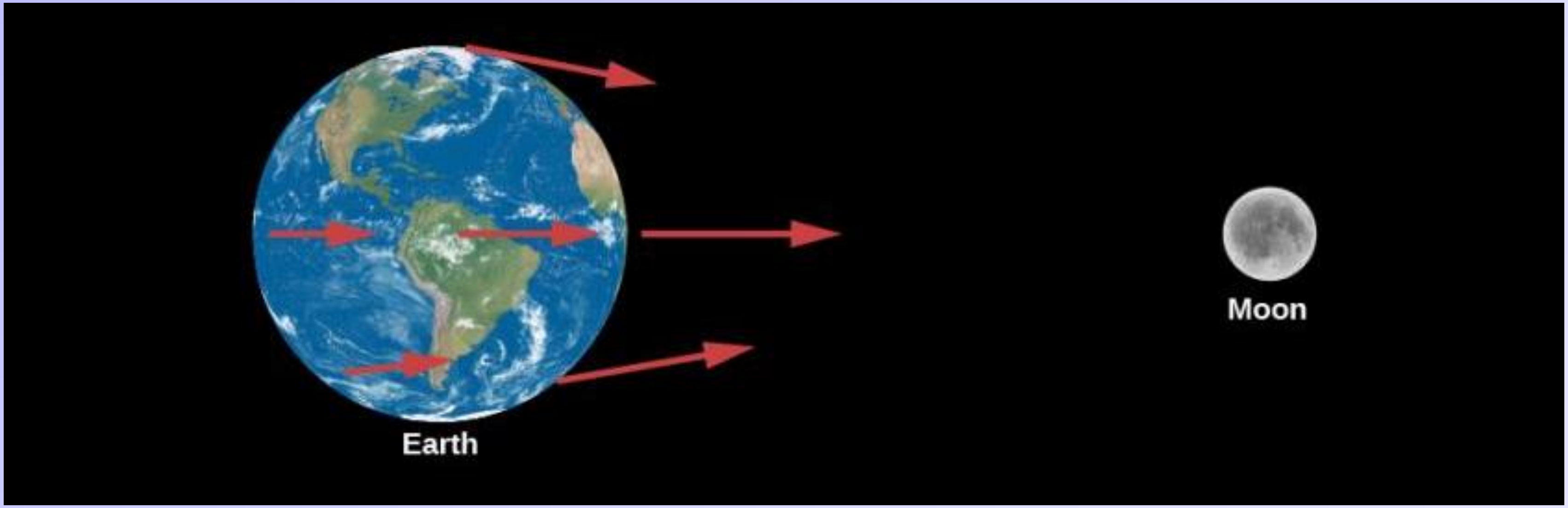
$$v_{escape}^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max\to\infty}}\right) \Rightarrow \boxed{v_{escape} = \sqrt{\frac{2GM_E}{R_E}}}$$

Ocean Tides

The force of **gravity** from the Moon is **not uniform** over the Earth.

→ gravity from Moon falls off as $1/r^2$.

→ Near face of Earth feels a stronger force than far face.

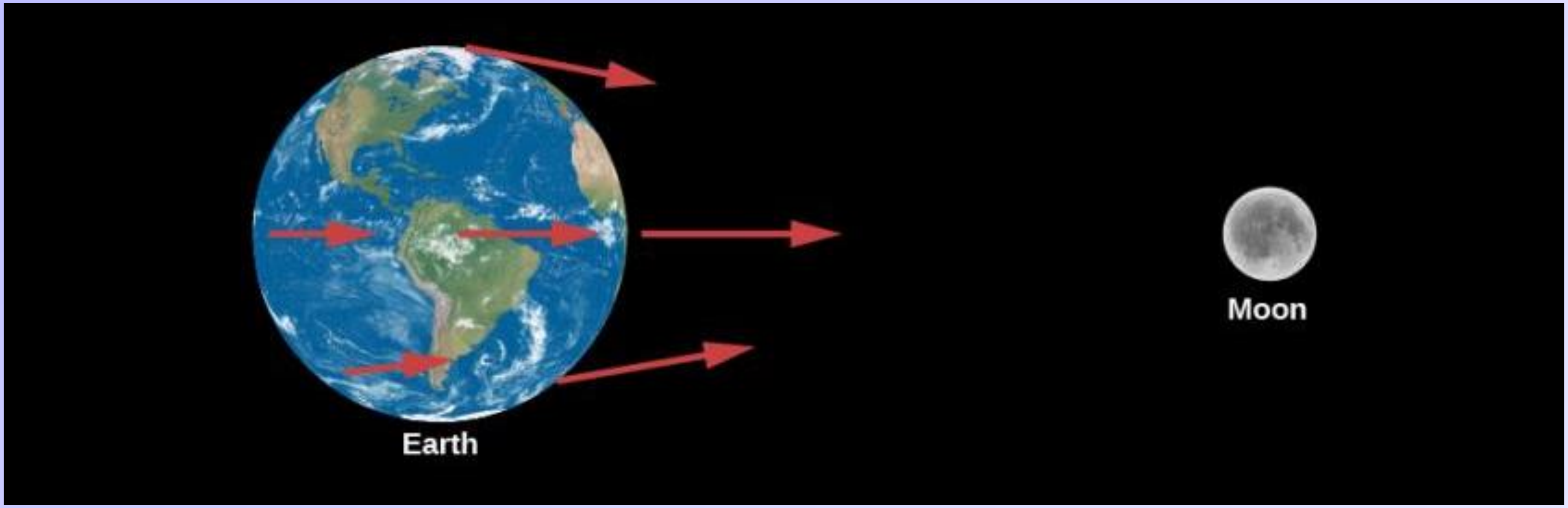


Ocean Tides

The force of **gravity** from the Moon is **not uniform** over the Earth.

→ gravity from Moon falls off as $1/r^2$.

→ Near face of Earth feels a stronger force than far face.



Result

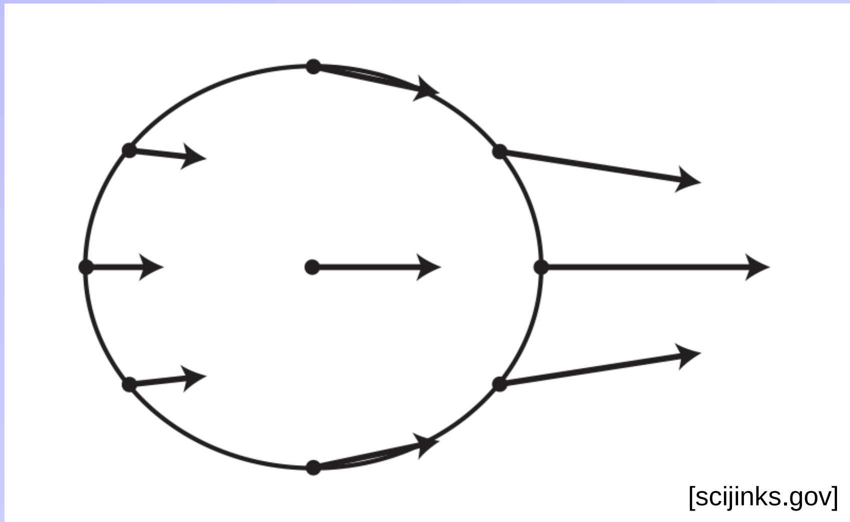
Water on **near side is pulled** towards Moon **more** than average Earth.

Water on **far side is pulled** towards Moon **less** than average Earth.

Ocean Tides: Effective Moon Gravity

Recall:

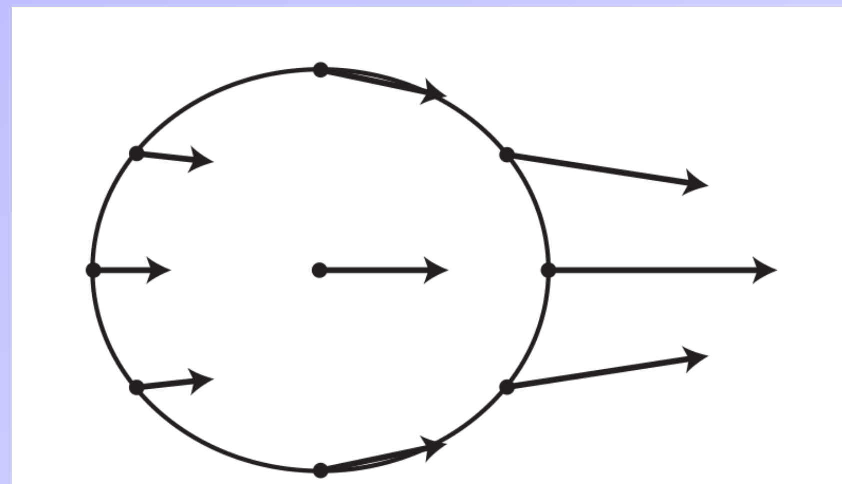
- Moon is in “free fall” orbit around Earth.
- Earth is in “free fall” orbit around Moon (albeit small orbit).



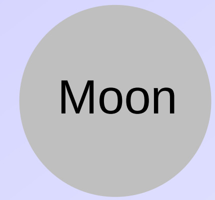
Ocean Tides: Effective Moon Gravity

Recall:

- Moon is in “free fall” orbit around Earth.
- Earth is in “free fall” orbit around Moon (albeit small orbit).

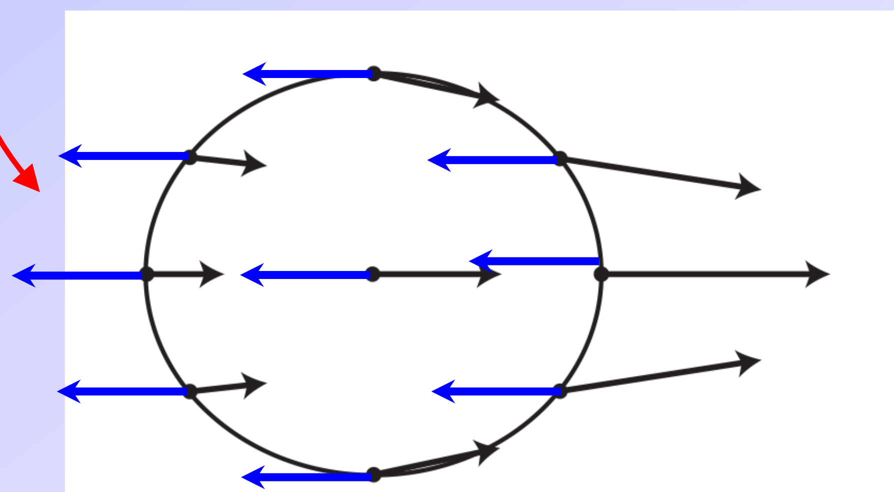


[scijinks.gov]



Subtract average gravitational force of Moon.

[since Earth is in “free fall” around Moon.]

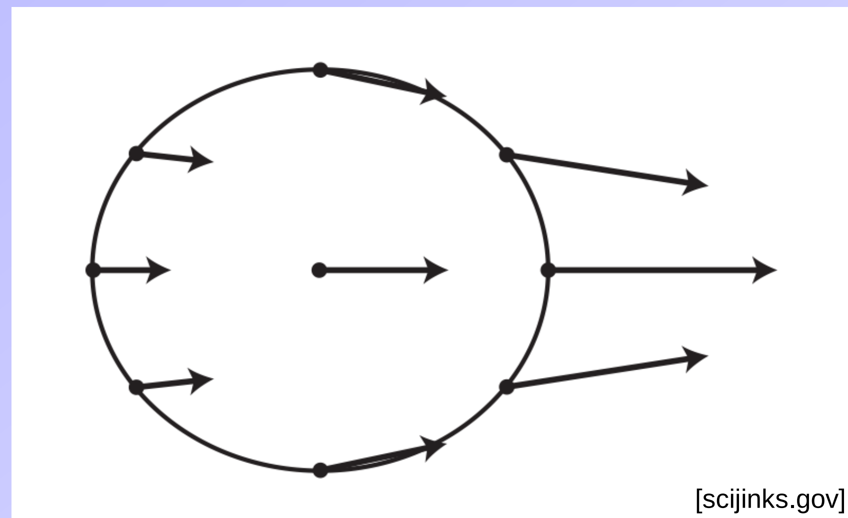


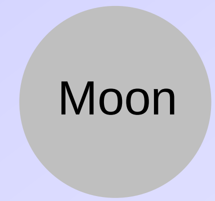
[scijinks.gov]

Ocean Tides: Effective Moon Gravity

Recall:

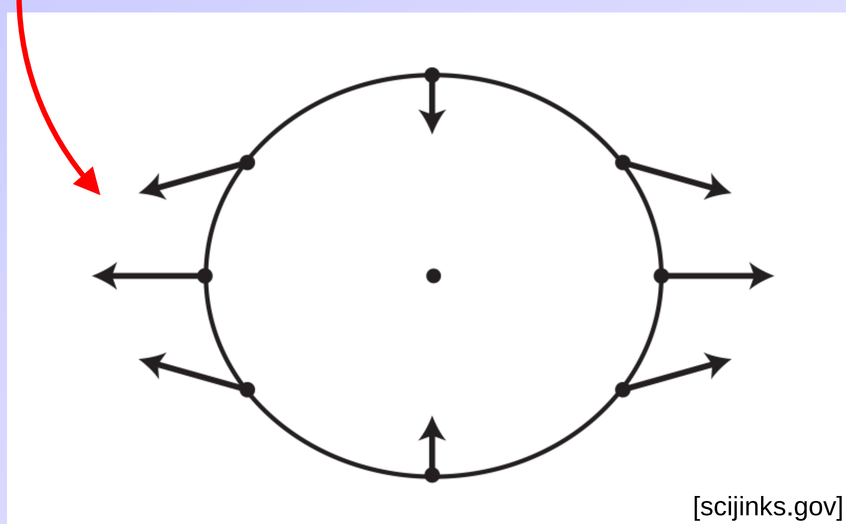
- Moon is in “free fall” orbit around Earth.
- Earth is in “free fall” orbit around Moon (albeit small orbit).





Subtract average gravitational force of Moon.

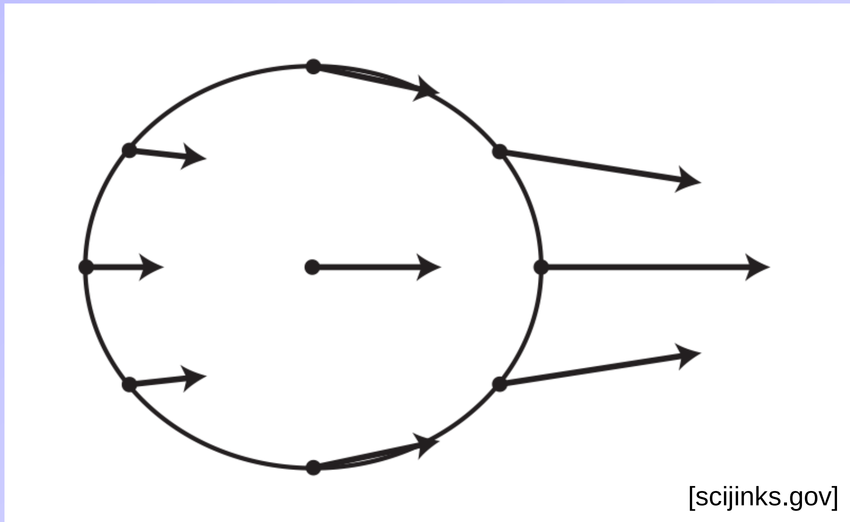
[since Earth is in “free fall” around Moon.]



Ocean Tides: Effective Moon Gravity

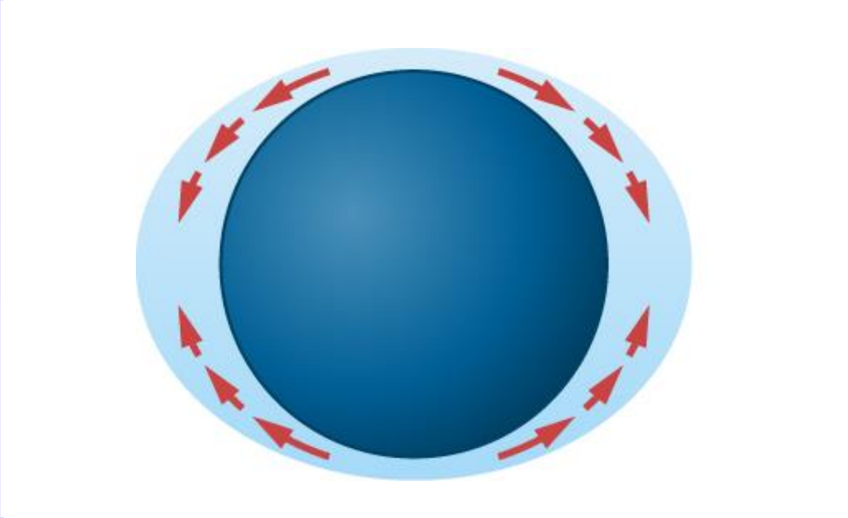
Recall:

- Moon is in "free fall" orbit around Earth.
- Earth is in "free fall" orbit around Moon (albeit small orbit).

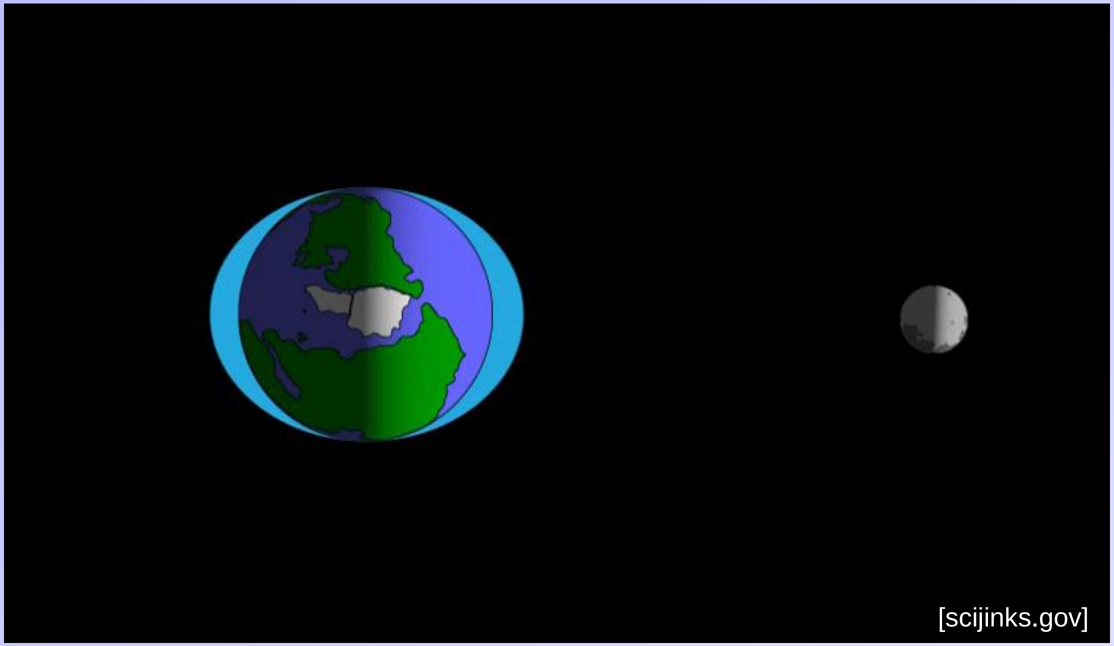


Moon

Ocean water is pulled by the effective force



Ocean Tides



Animation of Earth and Oceans as seen from above North Pole.

Sun's gravity gradient affects tides as well: 46% of Moon's contribution.

- Tides are largest when Sun-Moon-Earth are aligned.
- Tides are weakest when Sun & Moon are at 90° to each other.
- Shape of ocean basins & winds also affect the strength of tides.
- The atmosphere also experiences tides.