Today's Topics

Monday, August 31, 2020 (Week 3, lecture 6) – Chapter 3.

A. Momentum & energy

B. Gravity *by Newton*

C. Circular Motion

… Newton's version of Kepler's 3rd law.

Newton's Laws

of Classical Mechanics

1st Law: An object moves at constant velocity if there is no net force acting on it.

[**fine print:** in an inertial reference frame]

2nd Law: Force = mass $\times$ acceleration.

3rd Law: For any force, there is always an equal and opposite reaction force.

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- Conservation of ***Momentum***.
- Conservation of ***Energy***.

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total momentum
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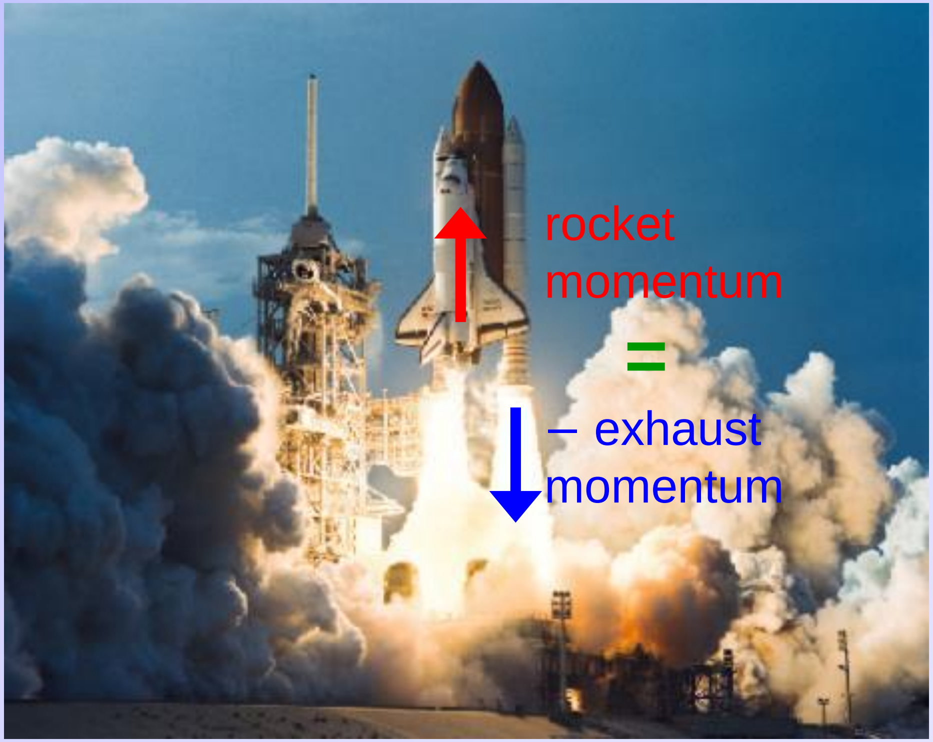
Conservation Law
The total momentum of a closed system never changes.

no external objects enter
no external forces

Momentum Conservation: Rocket Thrust

$$\text{Momentum}_{\text{rocket}} + \text{Momentum}_{\text{exhaust}} = 0$$



Conservation of Energy

Kinetic Energy $= E_k = \frac{1}{2}mv^2$

m = mass
v = speed

Potential Energy = "stored" energy

example: gravitational potential energy

Total Energy
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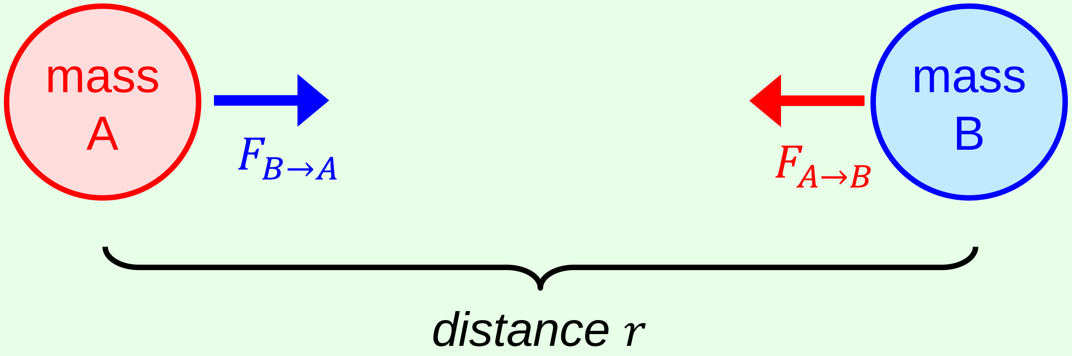
Newton's law of universal gravitation

All masses attract each other according to the following relation:

$$F_{A\rightarrow B} = -\mathrm{G}\frac{M_A M_B}{r^2} = -F_{B\rightarrow A}$$



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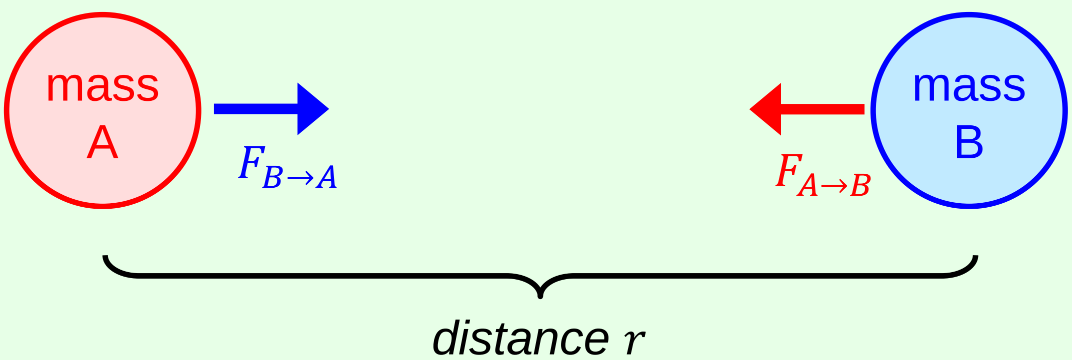


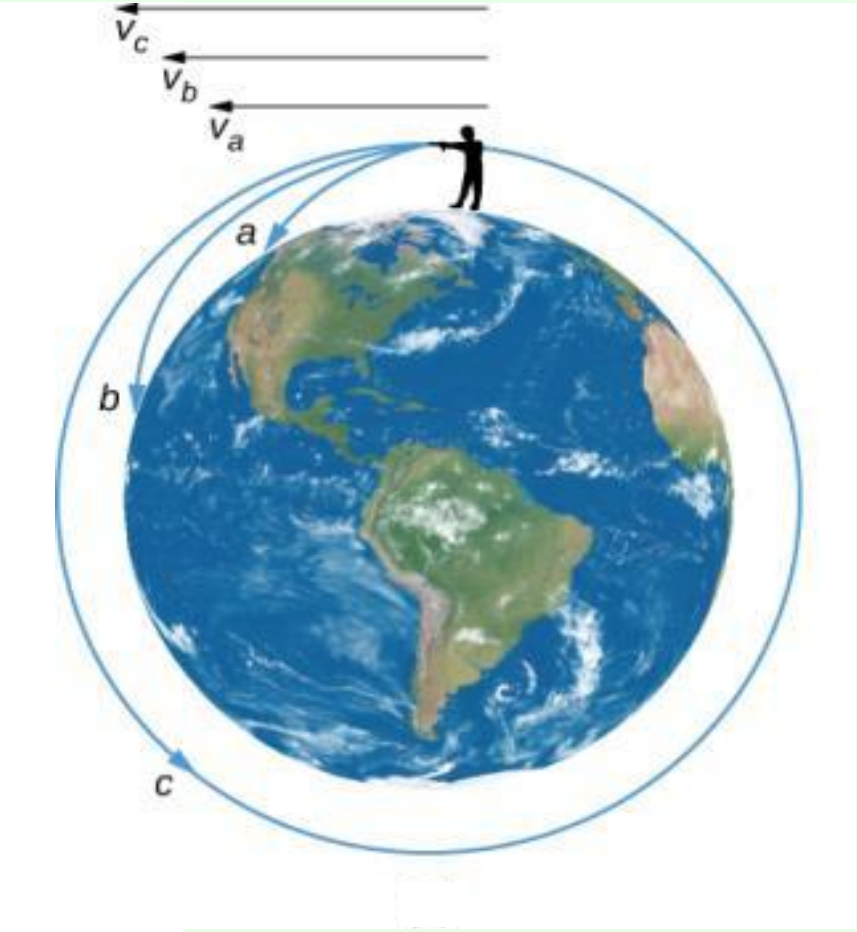
Properties

- Falls off as $1/r^2$.
- Proportional to M_A.
- Proportional to M_B.
- $G =$ Newton's constant $= 6.67430(15) \times 10^{-11}\ m^3/Kg\cdot s^2$

Why do all objects fall at the same rate?

(to be covered in problem session)

Orbiting is free falling while missing Earth



[OpenStax: Astronomy]

Paths a & b: Initial speeds are weak enough that Earth's gravity pulls the projectile back to the surface.

Path c: Initial speed is strong enough that Earth's gravity never pulls the projectile back to the surface.

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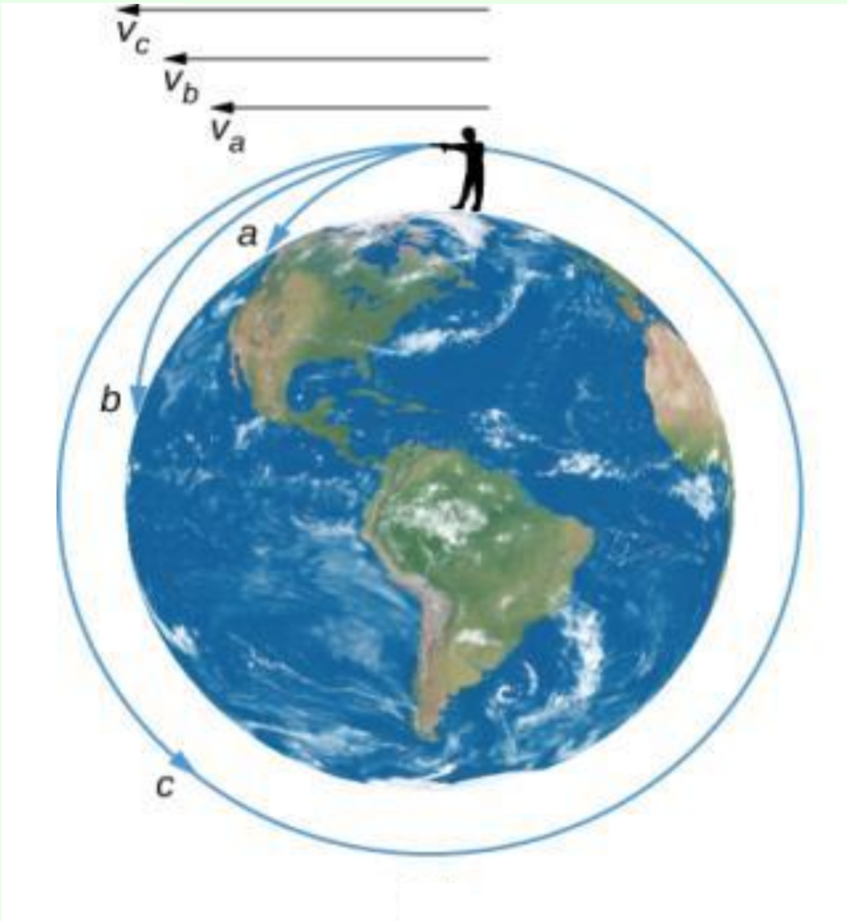

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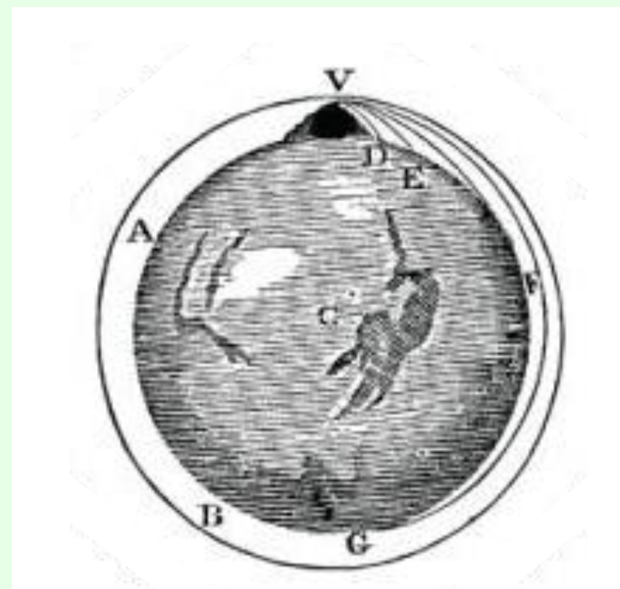


[Adapted from *De Mundi Systemate*, Newton (1731)] orbiting

"The knack of flying is learning how to throw yourself at the ground and miss"

- Hitchhikers Guide to the Galaxy

Weightless in Orbit

Clockwise from top left: Tracy Caldwell Dyson (NASA), Naoko Yamzaki (JAXA), Dorothy Metcalf-Lindenburger (NASA), and Stephanie Wilson (NASA). (credit: NASA)

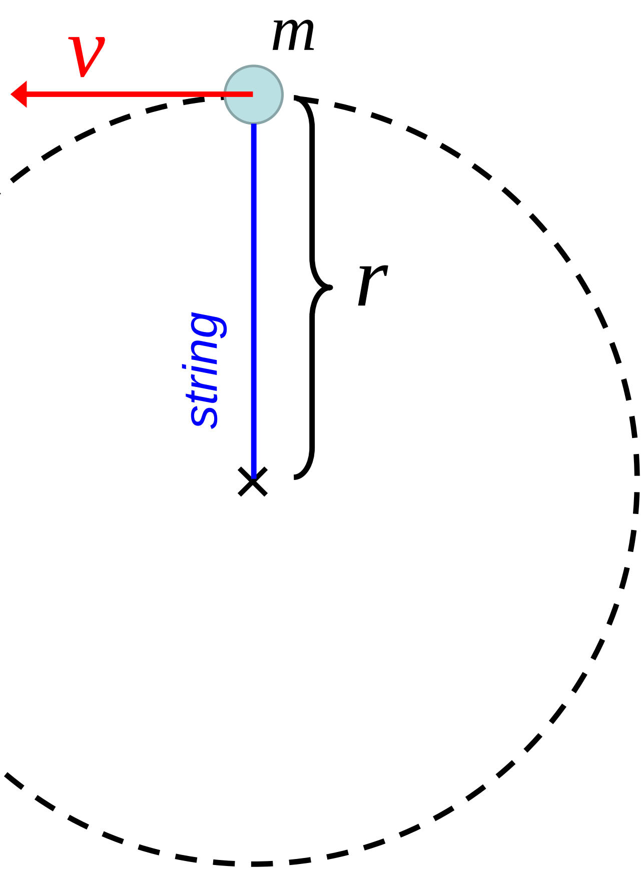
Astronauts in Free Fall: While in space, astronauts are falling freely, so they experience “weightlessness.”

Circular Motion

<u>Recall</u>

acceleration = **change** in $\underbrace{\text{velocity}}_{\textit{speed \& direction}}$ over time

Rotation is a type of <u>acceleration</u> where the velocity **direction changes**, but speed is constant.



“ball on a string”

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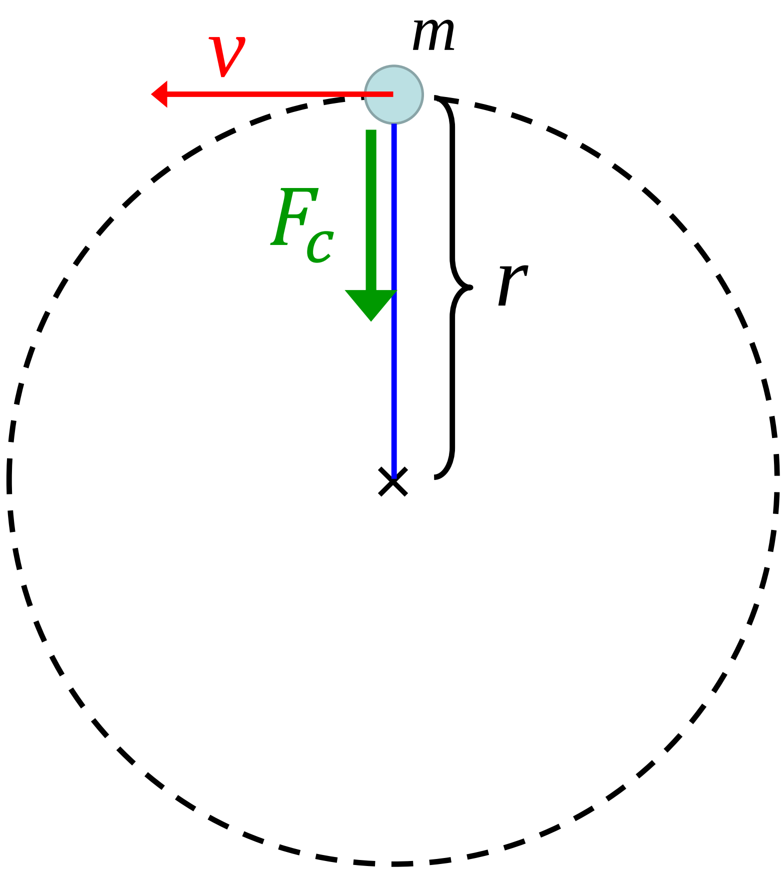

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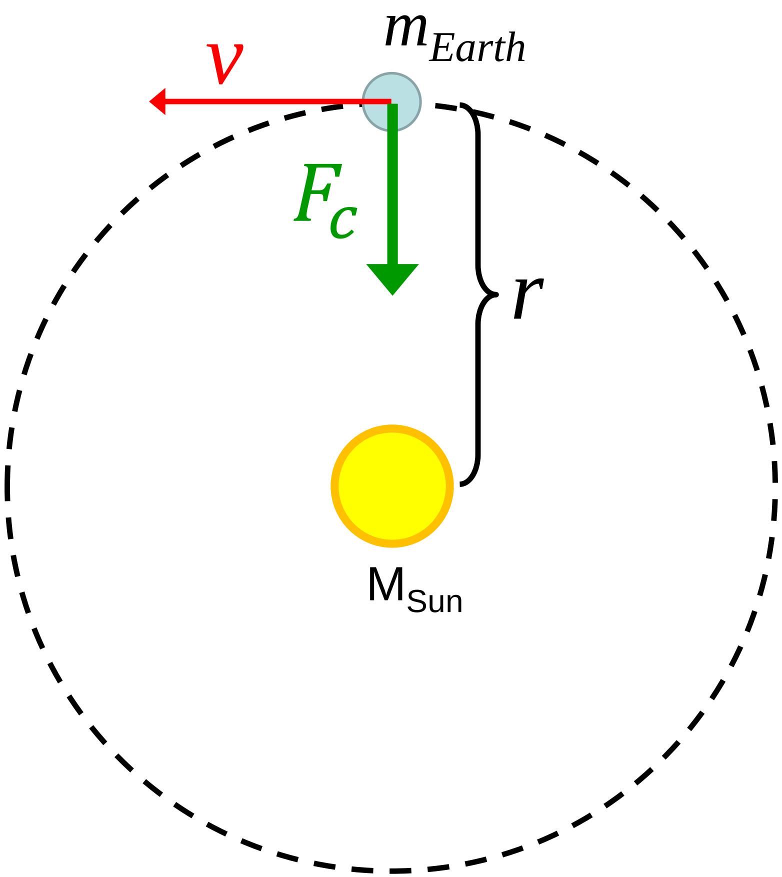
Acceleration: $a_c = \dfrac{v^2}{r}$

Centripetal Force: $F_c = \dfrac{mv^2}{r}$

Circular Motion Example: Earth's orbit of Sun

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$



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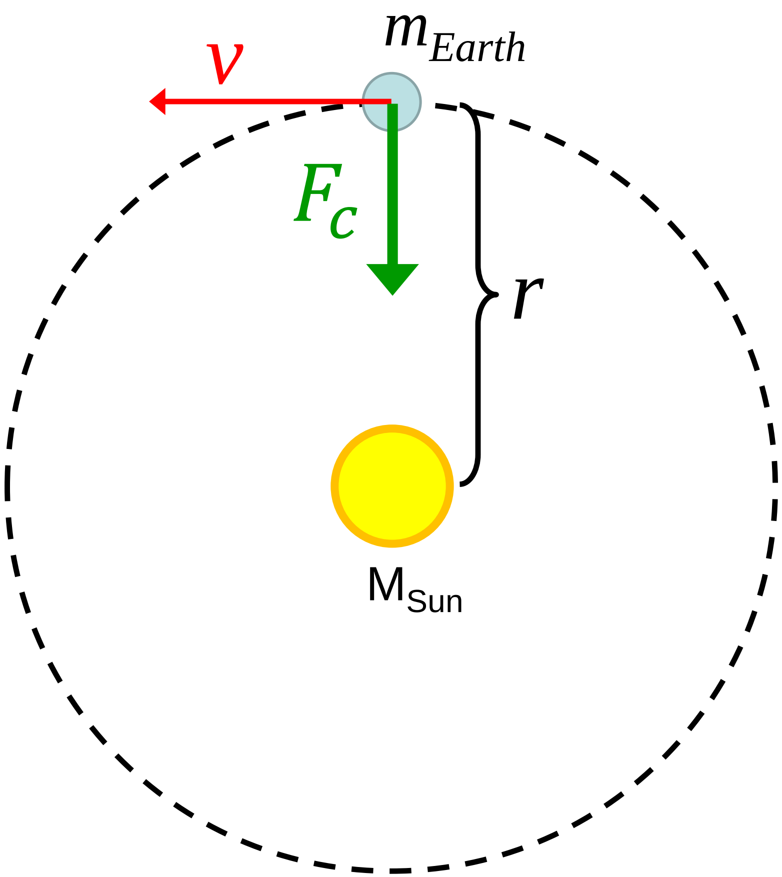

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The **centripetal force** that pulls on Earth to make it orbit the Sun **is gravity:**

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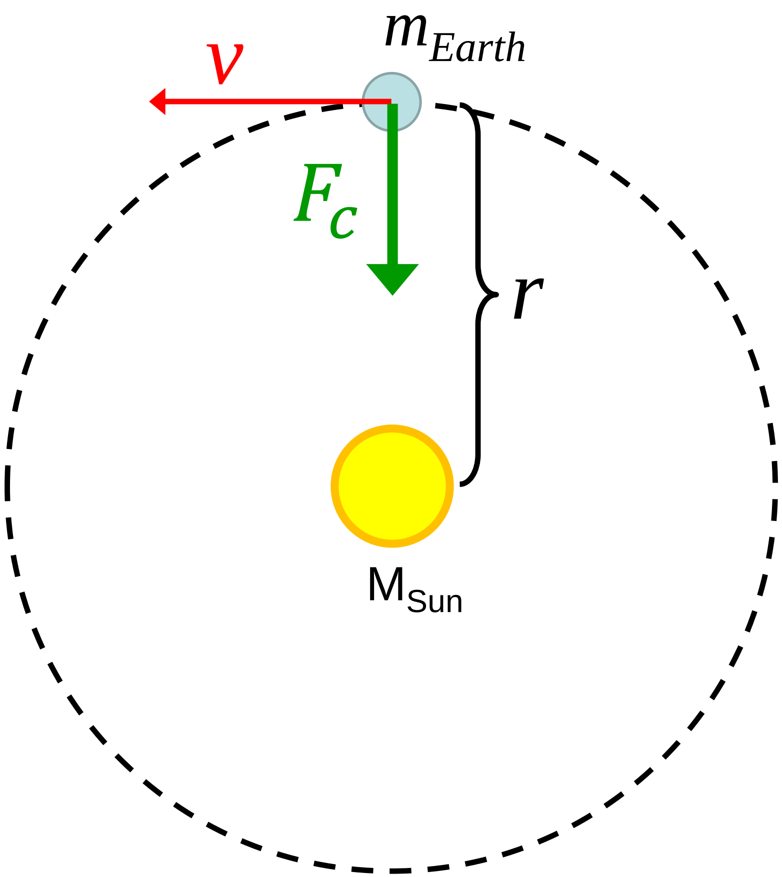

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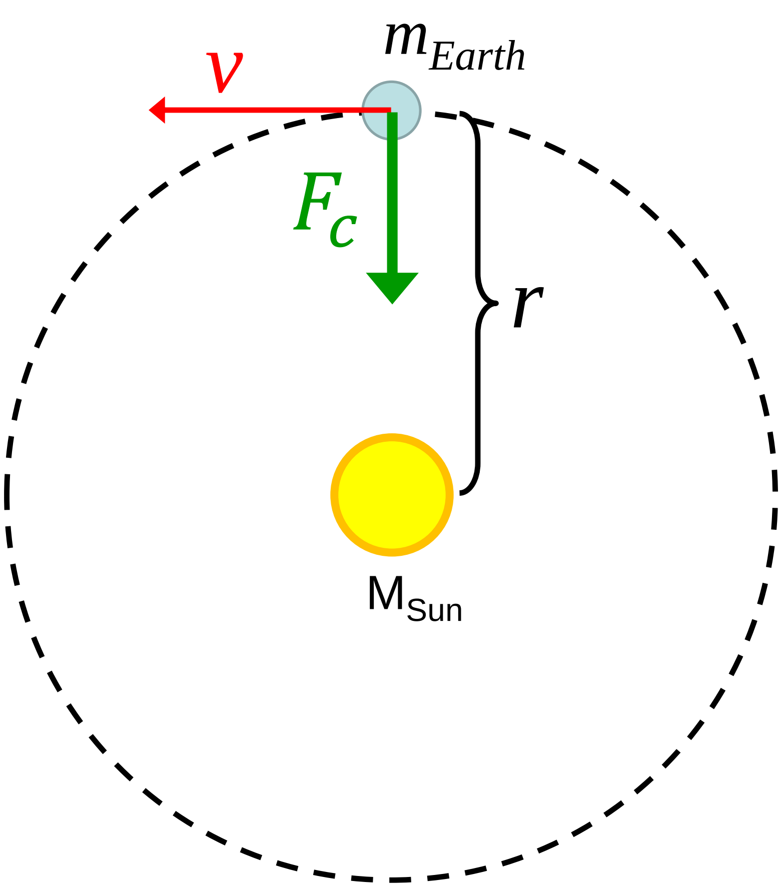

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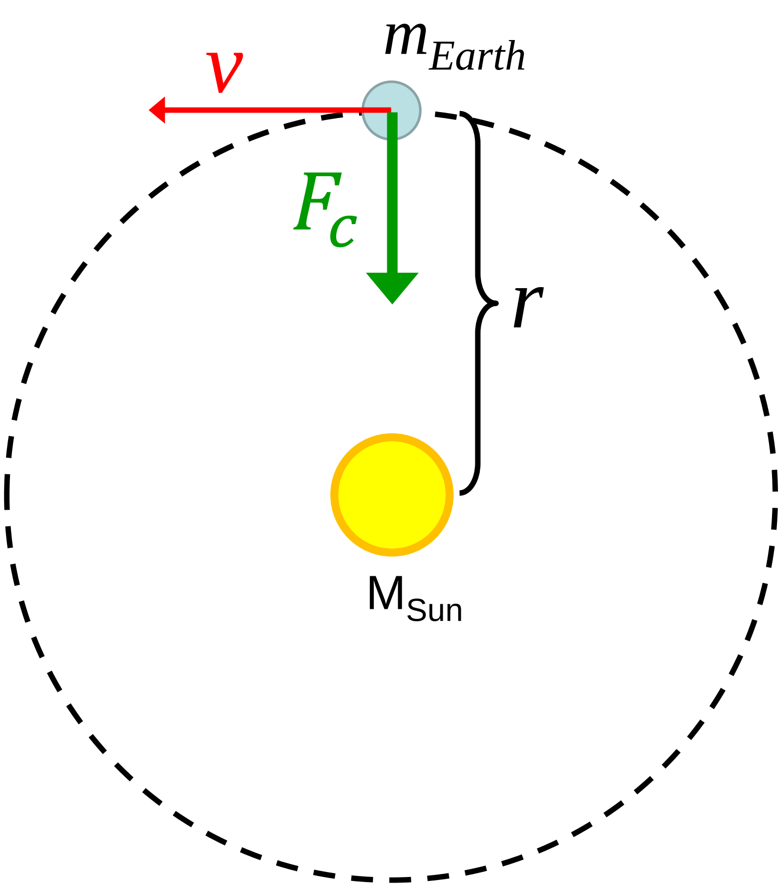

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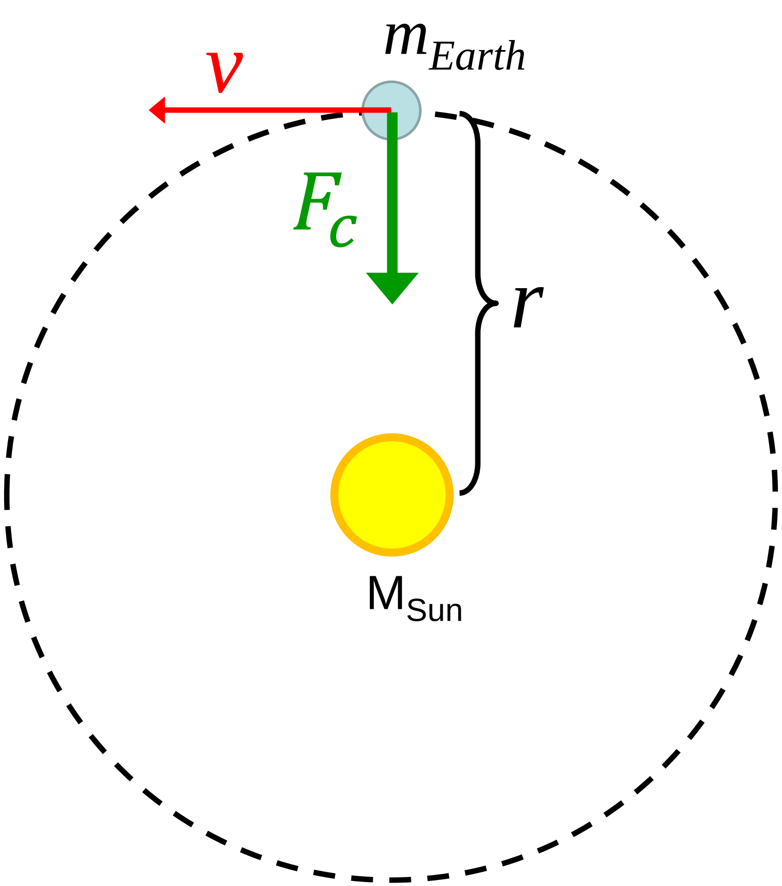

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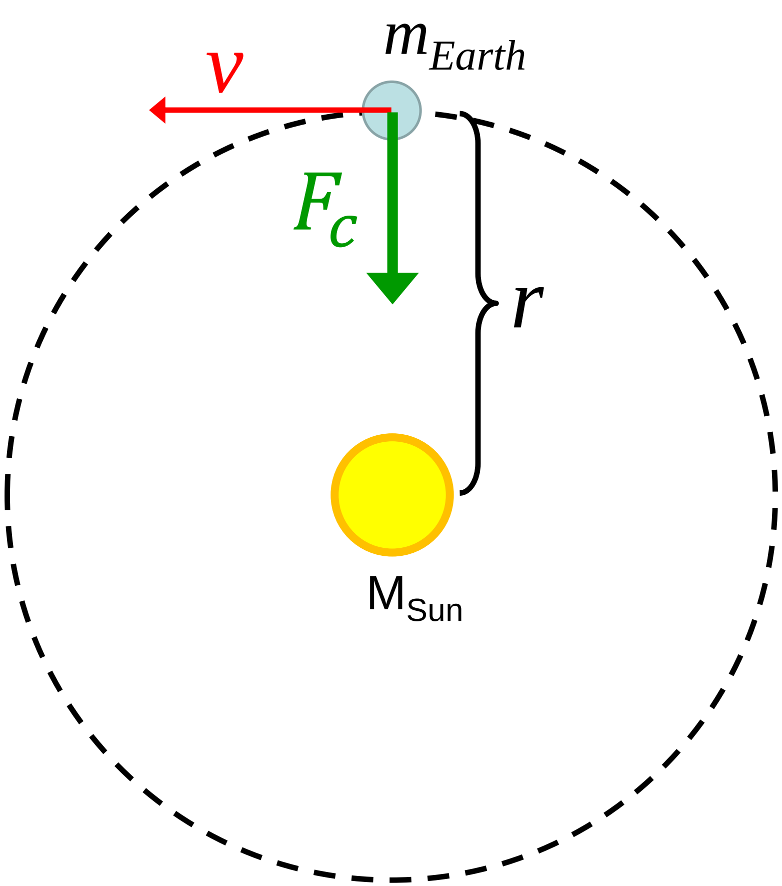
$v_{Earth} = 29.78 \times 10^3$ m/s

$r = 1\, AU = 149.6 \times 10^9$ m

$G = 6.67430(15) \times 10^{-11}\, m^3/Kg \cdot s^2$



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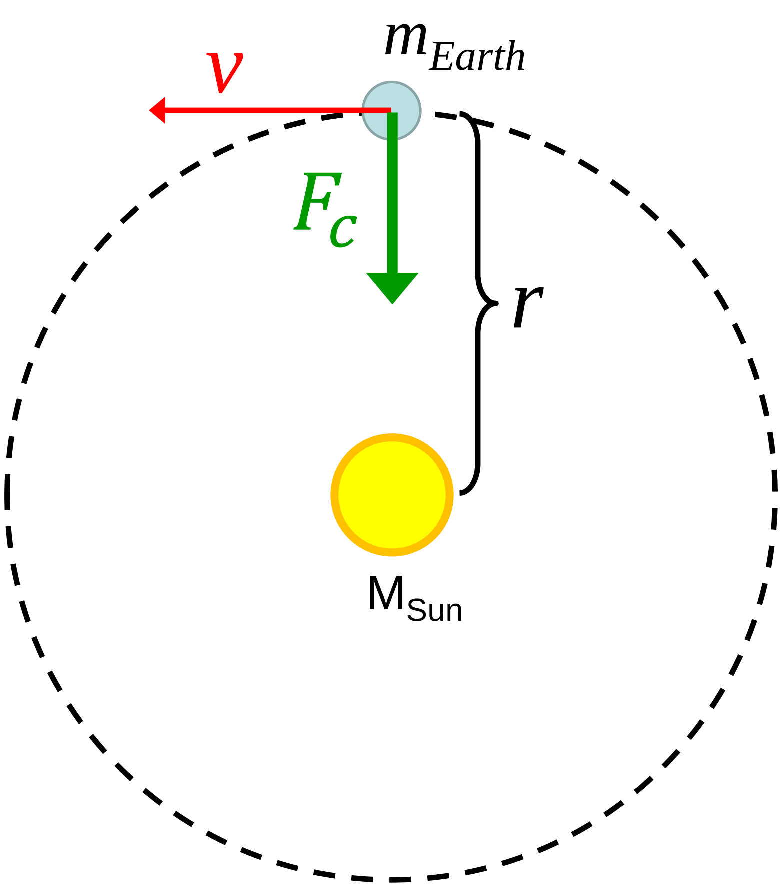
$$\boxed{M_{Sun} = 1.988 \times 10^{30} \text{ Kg}}$$



You can get the mass of the Sun from Earth's orbital parameters !!!

Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

Formula is in SI units

T = orbital period in seconds

a = semimajor axis in meters

$M_{1,2}$ =Mass of orbiting objects in Kg

G = 6.6743×10^{-11} $m^3/Kg.s^2$