

Friday, September 18, 2020

Example 1: Size of Betelgeuse

ALMA measured an angular size of $\theta = 0.05''$ at 640 ly
what's the diameter of Betelgeuse?

$$\begin{aligned} \text{angular size } \theta &= 0.05'' = \frac{0.05}{3600} \text{ degrees} = 1.39 \times 10^{-5} \text{ degrees} \\ &= 1.39 \times 10^{-5} \left(\frac{2\pi}{360} \right) \text{ rads} \\ &= 2.42 \times 10^{-7} \text{ rads} \end{aligned}$$

$$\begin{aligned} \text{distance} &= 640 \text{ ly} = 640 \text{ ly} \times \underbrace{64.2 \times 10^3 \frac{\text{AU}}{\text{ly}}}_{1 \text{ ly} = 64.2 \times 10^3 \text{ AU}} = 4.04 \times 10^7 \text{ AU} \end{aligned}$$



$$\frac{D}{d} = \frac{\text{rise}}{\text{run}} = \tan \theta = \tan (2.42 \times 10^{-7} \text{ rads}) = 2.42 \times 10^{-7}$$

$\tan(\theta_{\text{rad small}}) \approx \theta_{\text{rad small}}$

$$\begin{aligned} D &= d \tan \theta = (4.04 \times 10^7) (2.42 \times 10^{-7}) \\ &= 9.78 \text{ AU} \end{aligned}$$

\Rightarrow Betelgeuse has a diameter of $D = 9.78 \text{ AU}$