

Monday, September 21, 2020

#1

Example 1: Event Horizon Telescope's angular resolution

$$\begin{aligned}\text{Diameter} &= \text{Diameter of Earth} = 2 \times R_E \\ &= 12.7 \times 10^3 \text{ km} \\ &= 12.7 \times 10^6 \text{ m}\end{aligned}$$

$$\begin{aligned}\lambda &= 1.3 \text{ mm} = 1.3 \times 10^6 \text{ nm} \\ &= 1.3 \times 10^{-3} \text{ m} \\ &= 10^9 \text{ nm}\end{aligned}$$

$$\theta_{\text{min}} = 1.22 \frac{\lambda}{D}$$

radians

λ in m

D in m

or

$$\begin{aligned}\theta_{\text{min, arcseconds}} &= 0.000252 \frac{\lambda_{\text{nm}}}{D_{\text{m}}} \\ &= (0.000252) \frac{(1.3 \times 10^6 \text{ nm})}{(12.7 \times 10^6 \text{ m})} \\ &= 0.000026'' \\ &\quad \uparrow \quad \uparrow \\ &\quad \mu\text{as} \quad \mu\text{as}\end{aligned}$$

$$\Rightarrow \boxed{\theta_{\text{min}} = 26 \mu\text{as}}$$

Example #2: Density of the Earth

$$\text{Diameter} = 12,756 \text{ km} \Rightarrow \text{Radius} = \frac{\text{Diameter}}{2}$$

$$= \frac{12,756}{2}$$

$$\Rightarrow R_E = 6,378 \text{ km}$$

$$\text{Mass} = 59.8 \times 10^{23} \text{ kg}$$

$$\text{Volume of Earth} \approx \text{Volume of sphere}$$

$$= \frac{4}{3} \pi R_E^3$$

$$= \frac{4(3.1415926)(6378)^3}{3}$$

$$= 1.087 \times 10^{12} \text{ km}^3$$

$$\rho_E = \text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{59.8 \times 10^{23} \text{ kg}}{1.087 \times 10^{12} \text{ km}^3}$$

$$= 5.50 \times 10^{12} \text{ kg/km}^3$$

$$= 5.50 \times 10^3 \text{ kg/m}^3 \text{ (SI units)}$$

$$= 5.50 \text{ g/cm}^3$$