

Friday, September 25, 2020

Example: Potassium-40 decay

Q: How long does it take for 75% of  $^{40}\text{K}$  to decay?

A: fraction of remaining  $^{40}\text{K} = x_{\text{K}40} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$

25%

take natural log of both sides

$$\Rightarrow 0.25 = \left(\frac{1}{2}\right)^{t/t_{1/2}} \Rightarrow \ln(0.25) = \ln\left(\left(\frac{1}{2}\right)^{t/t_{1/2}}\right)$$

$$\Leftrightarrow \ln(0.25) = \left(\frac{t}{t_{1/2}}\right) \ln\left(\frac{1}{2}\right)$$

$$\Leftrightarrow \frac{\ln(0.25)}{\ln(0.5)} = \frac{t}{t_{1/2}} \Leftrightarrow t = t_{1/2} \frac{\ln(0.25)}{\ln(0.5)}$$

$$\Rightarrow t = (1.25 \times 10^9) \frac{(-1.38629)}{(-0.69315)} = 2$$

$$\Rightarrow t = (1.25 \times 10^9 \text{ years}) \times 2$$

$$\Rightarrow t = 2 \times (1.25 \times 10^9 \text{ years}) = 2 \text{ half-lives} \\ = 2.5 \text{ billion years}$$