

Wednesday, September 30, 2020

Example: Mass of the moon from its radius

$$\begin{aligned} R_{\text{Moon}} &= 27\% R_{\text{Earth}} \\ &= 0.27 R_{\text{Earth}} \end{aligned}$$

Volume:  $V_{\text{Moon}} = \frac{4}{3} \pi R_{\text{Moon}}^3$

$$V_{\text{Earth}} = \frac{4}{3} \pi R_{\text{Earth}}^3$$

$$\frac{V_{\text{Moon}}}{V_{\text{Earth}}} = \frac{\cancel{\left(\frac{4}{3}\right)\pi} R_{\text{Moon}}^3}{\cancel{\left(\frac{4}{3}\right)\pi} R_{\text{Earth}}^3} = \left(\frac{R_{\text{Moon}}}{R_{\text{Earth}}}\right)^3 = \left(\frac{0.27 R_{\text{Earth}}}{R_{\text{Earth}}}\right)^3$$

$$\begin{aligned} &= (0.27)^3 \\ &= 0.01968 \\ &\approx 0.02 \end{aligned}$$

$$\Rightarrow \boxed{\frac{V_{\text{Moon}}}{V_{\text{Earth}}} = 0.02} \quad (\Rightarrow) \quad V_{\text{Moon}} = 2\% V_{\text{Earth}}$$

For a same mass density, we expect the Moon to have 2% of the mass of Earth:  $M_{\text{Moon}} \Big|_{\text{estimate}} \approx 0.02 M_{\text{Earth}} \approx 1.2 \times 10^{23} \text{ kg}$

Note: Actually, the density of the Moon is less than Earth

$$M_{\text{Moon}} \Big|_{\text{actual}} = 0.734 \times 10^{23} \text{ kg}$$