Friday, October 2, 2020

Example: Temperature of the Earth surface without Greenhouse effect

Intensity of sunlight: $I = 1390 \, W/m^2$

- About $1/3$ of the light is reflected back into space.
- the rest of the light $(2/3)$ is absorbed by the planet.

$\Rightarrow$ Absorbed intensity: $I_{abs} = \frac{2}{3} \times 1390 \, W/m^2$
$927 \, W/m^2$

Total absorbed power: $\rightarrow$ the Earth makes a shadow disk with radius of Earth

$$\Rightarrow P_{absorbed} = I_{abs} \, \pi R_E^2$$
$$= (927 \, W/\cancel{m^2})(3.1415926)(6371 \times 10^3 \, \cancel{m})^{\cancel{2}}$$
$$= 1.182 \times 10^{17} \, W$$

$$\Rightarrow \boxed{P_{absorbed} = 1.182 \times 10^{17} \, W}$$

[note: World power consumption $\simeq 1.5 \times 10^{13} \, W$]

Blackbody radiation emitted by Earth:

Luminosity $= \sigma T^4$ (by each m^2) $\Rightarrow$ Power emitted: $P_{emitted} = (4\pi R_E^2) \, \sigma T^4$

$$\Rightarrow \quad P_{emitted} = 4\,(3.1415926)(6371 \times 10^3)^2 (5.67 \times 10^{-8})\, T^4$$
$$= (2.892 \times 10^7)\, T^4 \quad \text{watts}$$

For thermal equilibrium: $P_{absorbed} = P_{emitted}$

$$\Rightarrow \quad 1.182 \times 10^{17} = (2.892 \times 10^7)\, T_{eq}^4$$

$$\Rightarrow \quad T_{eq}^4 = \frac{1.182 \times 10^{17}}{2.892 \times 10^7} = 4.087 \times 10^9\ [K^4]$$

$$\Rightarrow \quad T_{eq} = \sqrt[4]{4.087 \times 10^9} = (4.087 \times 10^9)^{1/4}$$

$$= 253\ K$$

$$\Rightarrow \quad \boxed{T_{eq} = 253\ K = \underbrace{-20\,^\circ C}_{\substack{253-273 \\ = -20\,^\circ C}}}$$