Wednesday, October 14, 2020

Example: How big a telescope do you need to see Saturn's rings? [ABC rings]

Radius of outer A ring: $R_A = 137{,}000$ km

Distance to Saturn at closest approach to Earth $= 9.5\,AU - 1\,AU$

$= 8.5\,AU$

$= 8.5 \times (149.6 \times 10^6\ km)$

$= 1.272 \times 10^9$ km

Angular size of A ring: $\theta_A = \dfrac{2R_A}{d_{Earth-Saturn}} = \dfrac{2(1.37 \times 10^5)}{1.272 \times 10^9}$

$= 0.0002155$ rads $\times \frac{180}{\pi}$

$= 0.012346$ degrees $\times 3600$

$= 44.4$ arcseconds

$= 44.4''$

If we want 10 angular pixels to see the rings, then we need a telescope with an angular resolution of

$$\theta_{min} = \frac{\theta_A}{10} = \frac{44.4}{10} = 4.44 \text{ arcseconds}$$

What diameter do we need for the ~~~~ primary mirror or lens?

$$\theta_{min,\,arcseconds} = 0.000252\,\frac{\lambda_{nm}}{D_m}$$

$$\Longleftrightarrow \quad D_m = 0.000252\,\frac{\lambda_{nm}}{\theta_{min,\,arcseconds}}$$

500 nm for optical light

$$= 0.000252\,\frac{500}{4.44}$$

$$= 0.0284\ m = 2.84\ cm$$

In order to "see" that Saturn has rings, you need a telescope with a diameter of at least $D = 2.8$ cm