

Monday, April 14, 2025

Example: Estimating the mass of the Milky Way inside the Sun's orbit.

~~M~~ The solar system (i.e., Sun) travels on a roughly circular orbit with radius $R = 26 \text{ kly}$, velocity $= 230 \text{ km/s}$ and a orbital period $T \approx 226 \text{ million years}$

Convert to SI units:

$$R = 26 \text{ kly} = 26 \times 10^3 \text{ light years} = 2.44 \times 10^{17} \text{ km}$$

$$\underline{9.4 \times 10^{12} \text{ km}} = 2.44 \times 10^{20} \text{ m}$$

$$\text{velocity} = v = 230 \text{ km/s} = 230 \times 10^3 \text{ m/s}$$

$$T = 226 \text{ million years} = 226 \times 10^6 \text{ years}$$

$$\approx \pi \times 10^7 \text{ s}$$

$$= 7.12 \times 10^{15} \text{ s}$$

Method 1: Newton's version of Kepler's 3rd law

$$T^2 = \frac{4\pi^2}{G(M_{\text{galaxy inside}} + M_{\text{sun}})} a^3$$

$\swarrow$ neglect $M_{\text{galaxy}} \gg M_{\text{sun}}$

semi-major axis = radius for a circular orbit

$$\Rightarrow T^2 = \frac{4\pi^2}{G M_{\text{galaxy inside}}} r^3 \quad (\Rightarrow) \quad M_{\text{galaxy inside}} = \frac{4\pi^2}{G} \frac{R^3}{T^2}$$

$$\begin{aligned}
 \Rightarrow M_{\text{galaxy inside}} &= \frac{4 (3.1415926)^2 (2.44 \times 10^{20})^3}{6.6743 \times 10^{-11} (7.12 \times 10^{15})^2} \\
 &= 1.695 \times 10^{41} \text{ kg} \\
 &= 8.52 \times 10^{10} M_{\text{sun}} \quad \left. \begin{array}{l} \div M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \end{array} \right\} \\
 &= 85 \text{ billion } M_{\text{sun}}
 \end{aligned}$$

Method 2: Gravitational force

Force of gravity: $F_g = \frac{G M_{\text{galaxy}} M_{\text{sun}}}{R^2}$

Centripetal force for circular motion: $F_c = \frac{M_{\text{sun}} v^2}{R}$

$$F_g = F_c \Leftrightarrow \frac{G M_{\text{galaxy}} \cancel{M_{\text{sun}}}}{R^2} = \frac{\cancel{M_{\text{sun}}} v^2}{R}$$

$$\Leftrightarrow M_{\text{galaxy}} = \frac{R v^2}{G} = \frac{(2.44 \times 10^{20})(230 \times 10^3)^2}{6.6743 \times 10^{-11}}$$

$$\begin{aligned}
 &= 1.934 \times 10^{41} \text{ kg} \\
 &= 9.72 \times 10^{10} M_{\text{sun}} \quad \left. \begin{array}{l} \div M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \end{array} \right\} \\
 &= 97 \text{ billion } M_{\text{sun}}
 \end{aligned}$$