

Monday, April 21, 2025

Example: Determining distance with a Cepheid Variable star

a) Consider a type I Cepheid variable star with period = 4 days.

↳ Q: What is its intrinsic luminosity?

↳ A: From plot $L_{\text{Cepheid}} \approx 1000 L_{\text{sun}} = 10^3 L_{\text{sun}}$

b) If we measure the Cepheid's apparent brightness to be 10 trillion times less than the Sun's, then how far away is it?

$I_{\text{apparent Cepheid}} =$ apparent intensity of Cepheid (as seen on Earth)

$$= \frac{L_{\text{Cepheid}}}{4\pi d_{\text{Cepheid}}^2} \quad ; \quad d_{\text{Cepheid}} = \text{distance to Cepheid}$$

$$I_{\text{apparent Sun}} = \frac{L_{\text{Sun}}}{4\pi d_{\text{Sun}}^2} \quad ; \quad d_{\text{Sun}} = \text{distance to Sun} = 1 \text{ AU}$$

$$\frac{I_{\text{apparent Cepheid}}}{I_{\text{apparent Sun}}} = \frac{1}{10 \text{ trillion}} = \frac{1}{10 \times 10^{12}} = \frac{1}{10^{13}} = \frac{\frac{L_{\text{Cepheid}}}{4\pi d_{\text{Cepheid}}^2}}{\frac{L_{\text{Sun}}}{4\pi d_{\text{Sun}}^2}}$$

$$\begin{aligned}
 \Rightarrow \frac{1}{10^{13}} &= \frac{\frac{L_{\text{Cepheid}}}{d_{\text{Cepheid}}^2}}{\frac{L_{\text{Sun}}}{d_{\text{Sun}}^2}} = \frac{L_{\text{Cepheid}}}{L_{\text{Sun}}} \times \frac{d_{\text{Sun}}^2}{d_{\text{Cepheid}}^2} \\
 &= \frac{L_{\text{Cepheid}}/L_{\text{Sun}}}{\left(\frac{d_{\text{Cepheid}}}{d_{\text{Sun}}}\right)^2} \quad \leftarrow 10^3 \quad \left(\frac{d_{\text{Cepheid}}}{d_{\text{Sun}}}\right)^2
 \end{aligned}$$

d_{Cepheid} in AU

$$\Rightarrow \frac{1}{10^{13}} = \frac{10^3}{(d_{\text{Cepheid, AU}})^2} \quad (\Rightarrow) d_{\text{Cepheid, AU}}^2 = \frac{10^3 \times 10^{13}}{10^{16}}$$

$$\Rightarrow d_{\text{Cepheid, AU}} = \sqrt{10^{16}} = (10^{16})^{1/2} = 10^8$$

$$\Rightarrow d_{\text{Cepheid, AU}} = 10^8 \text{ AU}$$

$\Rightarrow$ The distance to the Cepheid is $10^8 \text{ AU} = 100 \text{ million AU}$

$$\begin{aligned}
 d = 10^8 \text{ AU} &= 10^8 \times (149.6 \times 10^6 \text{ km}) = 149.6 \times 10^{14} \text{ km} \\
 &\div \frac{9.4 \times 10^{12} \text{ km}}{1 \text{ light year}} = 1591 \text{ light year} \\
 &\approx 1.6 \times 10^3 \text{ ly}
 \end{aligned}$$

$\Rightarrow$ The distance to the Cepheid is $10^8 \text{ AU} \approx 1.6 \times 10^3 \text{ light years}$