Wednesday, April 23, 2025

Example: Determining distance with a type 1a supernova.

Using the Hubble Space Telescope you observe a type 1a supernova that peaks in apparent brightness at 1000 photons/s (in the visible).

Q: How far away is the supernova?

Step 1: What is the apparent brightness in W/m^2?

$$\text{Diameter of } \overbrace{\text{Hubble Space Telescope}}^{\text{"HTS"}} = 2.4\ m$$

↳ $\text{Radius} = \frac{\text{Diameter}}{2} = \frac{2.4}{2} = 1.2\ m$

↳ $$\text{Area of HTS} = \pi\,\text{Radius}^2$$
$$= (3.1415)(1.2)^2$$
$$= 4.52\ m^2$$
$$\simeq 4.5\ m^2$$

$\Rightarrow \boxed{\text{Area of HTS} \simeq 4.5\ m^2}$

Next, Convert 1000 photons/s to $\underbrace{\text{"Energy" per second}}_{\text{Joules/s} = \text{Watts}}$

Middle of visible spectrum is $\lambda = 500\ nm$

$$\Rightarrow \lambda f = c \Leftrightarrow f = \frac{c}{\lambda} = \frac{3.0 \times 10^{8}\ m/s}{500 \times 10^{-9}\ m}$$

$$\Rightarrow f = 6.0 \times 10^{14}\ Hz$$

Energy of 1 photon is $E_\gamma = hf = (6.626\ 10^{-34}\ J{\cdot}s)(6.0 \times 10^{14}\ s^{-1})$

$\Rightarrow \quad E_\gamma = 3.976 \times 10^{-19}\ J$

$$\Rightarrow \quad 1000\ \text{photons/s} = \frac{1000 \times (3.976 \times 10^{-19}\ J)}{1\ s}$$

$$= 3.976 \times 10^{-16}\ \underbrace{J/s}_{\text{Watts}}$$

$$\Rightarrow \quad \boxed{1000\ \text{photons/s} = 3.976 \times 10^{-16}\ W} = \text{power}$$

$\Rightarrow$ Apparent brightness in W/m^2 is Intensity $= \frac{\text{Power}}{\text{Area}}$

$$\Rightarrow \quad \boxed{\text{Intensity} = \frac{3.976 \times 10^{-16}\ W}{4.5\ m^2} = 8.8355 \times 10^{-17}\ W/m^2}$$

Step 2: How far away is supernova?

From lecture 31 (slides 4-5): $L_{\substack{MAX\\1A}} = 5 \times 10^9\ L_{sun}$ = power (in visible light) (i.e. watts)

Note: $L_{sun} = 3.8 \times 10^{26}$ Watts

$$\Rightarrow \quad L_{\substack{max\\1A}} = (5 \times 10^9) \times (3.8 \times 10^{26}) = 1.9 \times 10^{36}\ \text{Watts}$$

Apparent brightness = Intensity $= \dfrac{L_{\substack{max\\1A}}}{4\pi d^2}$

[d = distance to Supernova] $\quad \Leftrightarrow \quad d^2 = \dfrac{L_{\substack{max\\1A}}}{4\pi \times \text{Intensity}}$

$$= \frac{1.9 \times 10^{36}}{4(3.1415)(8.8355 \times 10^{-17})}$$

$$= 1.7112 \times 10^{51}\ m^2$$

$$\Rightarrow \quad d = 4.137 \times 10^{25} \text{ m} = 4.137 \times 10^{22} \text{ km}$$

$$\div \underbrace{9.4 \times 10^{12} \text{ km}}_{1 \text{ light year}}$$

$$= 4.4 \times 10^{9} \text{ light years}$$

The supernova is in a galaxy at a distance of

$$\boxed{d = 4.4 \times 10^{9} \text{ light years} = 4.4 \text{ billion light years}}$$