Wednesday, April 30, 2025

**Example**: Critical density of the universe to bring its expansion to a halt at infinite time.
(scenario 3)

Einstein's equations of General Relativity lead to the following expression for the critical density

$$\rho_{critical} = \frac{3H^2}{8\pi G}$$

$H$ ← Hubble's constant

$G$ ← Newton's gravitational constant

$$G = 6.6743 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$H = \frac{22 \text{ km/s}}{\text{million-light years}} = \frac{22 \times 10^3 \text{ m/s}}{10^6 \times (9.4 \times 10^{12} \text{ km})}$$

($9.4 \times 10^{12}$ km $= 9.4 \times 10^{15}$ m)

$$= \frac{22 \times 10^3 \text{ m/s}}{10^6 \times 9.4 \times 10^{15} \text{ m}} = \frac{22 \times 10^3 \text{ m/s}}{9.4 \times 10^{21} \text{ m}}$$

$$= 2.3 \times 10^{-18} \text{ s}^{-1}$$

$$\Rightarrow \rho_{critical} = \frac{3}{8(3.1415)} \frac{\left(2.3 \times 10^{-18} \text{ s}^{-1}\right)^2}{6.6743 \times 10^{-11} \frac{m^3}{kg \cdot s^2}}$$

($(s^{-1})^2 \rightarrow \frac{1}{s^2}$)

$$= 9.46 \times 10^{-27} \frac{kg}{m^3}$$

$$\Rightarrow \boxed{\rho_{critical} = 9.46 \times 10^{-27} \frac{kg}{m^3} \simeq 10^{-26} \frac{kg}{m^3}}$$

$\Rightarrow$ Corresponds to a grain of dust (mass $= 10^{-17}$ kg) per cube 10 km on the side.