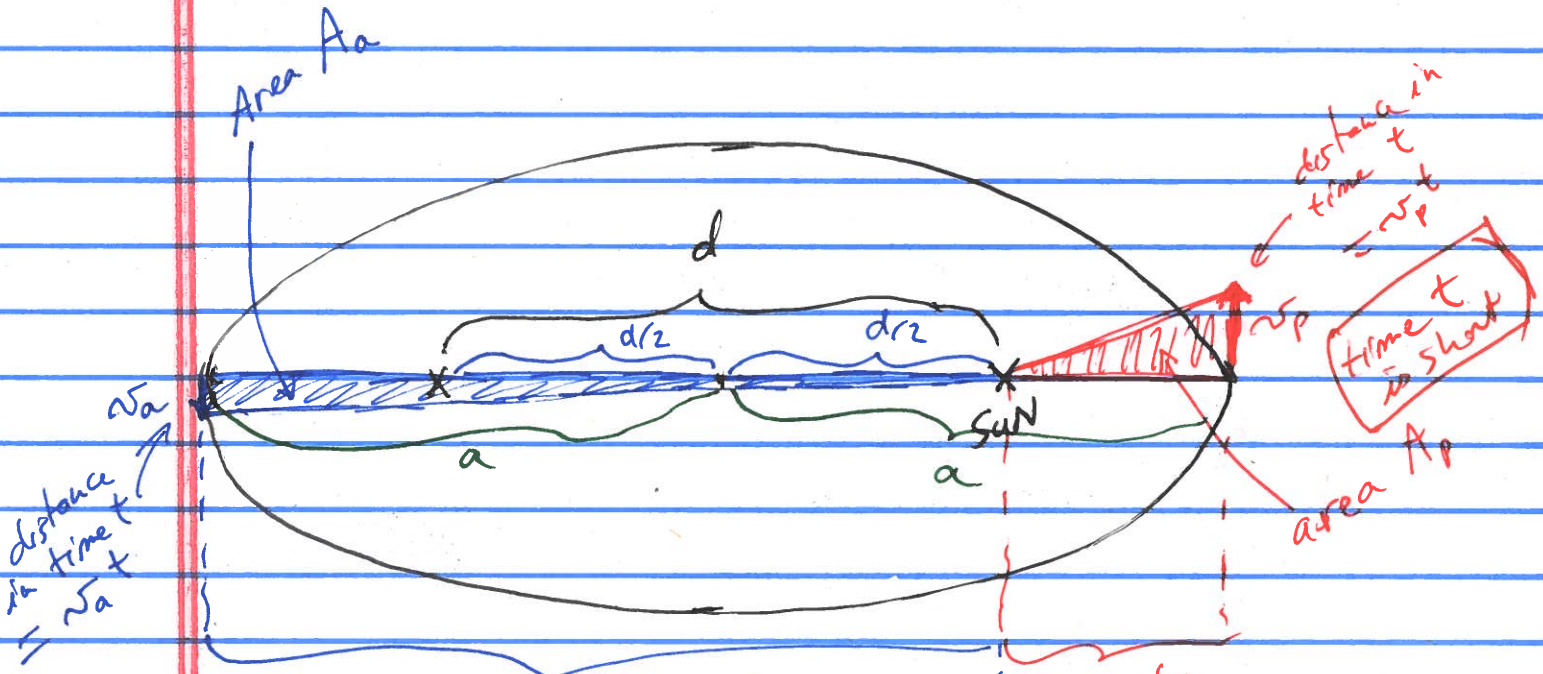
Monday, February 3, 2025 (Continued from Friday, Jan. 31)

Speeds at aphelion and perihelion

(far, slow) (near, fast)



Area of the aphelion triangle $= \frac{(v_a t)(a + d/2)}{2}$

Area of the perihelion triangle $= \frac{(v_p t)(a - d/2)}{2}$

$A_a = A_p$

Kepler's 2nd Law: Area of aphelion triangle = Area of the perihelion triangle

$$\frac{(v_a t)(a + d/2)}{2} = \frac{(v_p t)(a - d/2)}{2}$$

$$\Leftrightarrow \quad \boxed{\frac{v_p}{v_a} = \frac{a + d/2}{a - d/2}} = \frac{2a + d}{2a - d}$$

divide numerator & denominator by $2a$

multiply by $\frac{2}{2}$

$$\frac{v_p}{v_a} = \frac{\frac{2a}{2a} + \frac{d}{2a}}{\frac{2a}{2a} - \frac{d}{2a}}$$

($\frac{2a}{2a} \to 1$, $\frac{d}{2a} \to \varepsilon$, ε = eccentricity)

$$\Leftrightarrow \quad \boxed{\frac{v_p}{v_a} = \frac{1 + \varepsilon}{1 - \varepsilon}}$$

Example 1: Earth with $\varepsilon = 0.02$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.02}{1 - 0.02} = \frac{1.02}{0.98} \simeq 1.04$$

$\Rightarrow$ Earth orbital speed varies by 4%.

Example 2: Mars with $\varepsilon = 0.09$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.09}{1 - 0.09} \simeq 1.2$$

$\Rightarrow$ Mars's orbital speed varies by 20%.