Monday, February 3, 2025

Kepler's Laws. Applicability

Applies everywhere

1st Law: All orbits are ellipses, circles, parabolas, or hyperbolas

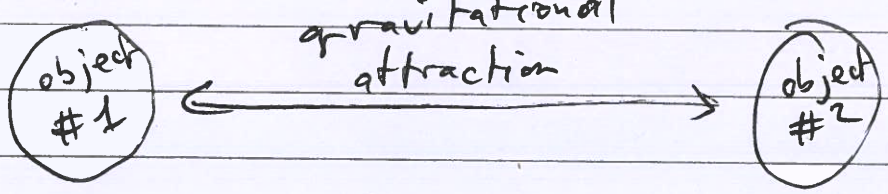
2nd Law: Law of equal areas

3rd Law: $T^2 = a^3$

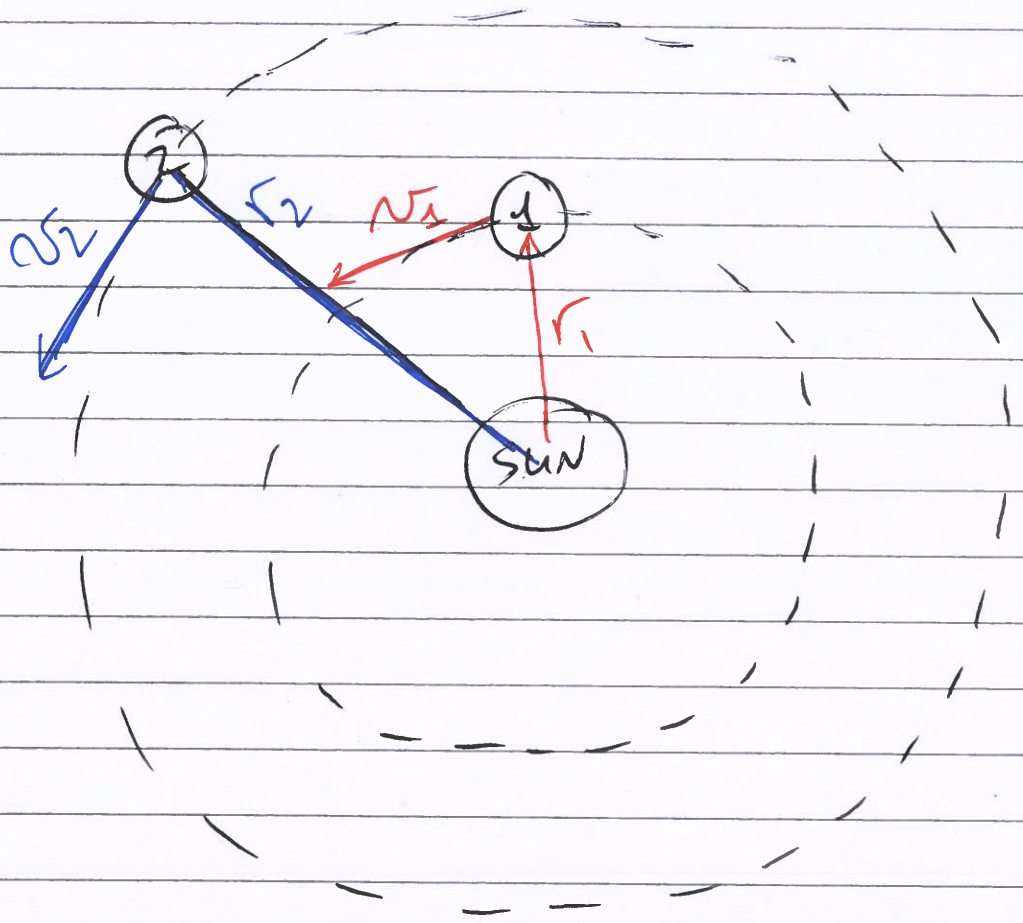
T = orbital period in Earth years

a = semimajor axis in A.U.

It "should" apply everywhere, but units only make it valid for our Solar System.



Orbital speed vrs Orbital distance



Assume circular orbits for simplicity

[good approximation for our solar system]

$$\rightarrow \begin{cases} r_1 = a_1 \\ r_2 = a_2 \end{cases}$$

Let's calculate the velocity / speed :

orbital circumference $= 2\pi r$

orbital period $= T = \dfrac{2\pi r}{v}$ (1) $\Leftrightarrow v = \dfrac{2\pi r}{T}$

orbital speed

Kepler's 3rd Law: $T^2 = a^3 = r^3$

$\Leftrightarrow T = r^{3/2} = \sqrt{r^3}$ (2)

Combine equations (1) & (2): $\frac{2\pi r}{v} = T = r^{3/2}$

$\Leftrightarrow \frac{2\pi r}{v} = r^{3/2}$ $\Leftrightarrow v = \frac{2\pi r}{r^{3/2}}$ $^{1/2}$

$\Leftrightarrow v = \frac{2\pi}{r^{1/2}}$

$\Leftrightarrow$ $\boxed{v = \frac{2\pi}{\sqrt{r}}}$

units of AU/yr

units of AU

Conclusion: The further out a planet is, the slower its speed.