Today's Topics

Friday, February 7, 2025 (Week 2, lecture 7) – Chapter 3.

0. Gravity review

1. Circular Motion

   … Newton's version of Kepler's 3rd law.

2. Center of Mass

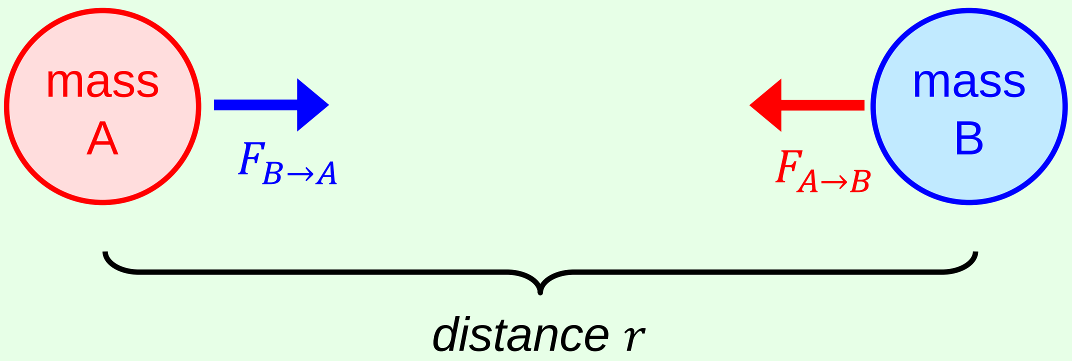
3. Angular momentum

Gravity Review

Newton's law of universal gravitation

All masses attract each other according to the following relation:

$$F_{A\to B} = -\mathrm{G}\frac{M_A M_B}{r^2} = -F_{B\to A}$$



Properties

- Falls off as $1/r^2$.
- Proportional to M_A.
- Proportional to M_B.
- G = Newton's constant $= 6.67430(15) \times 10^{-11}$ $m^3/Kg \cdot s^2$

Why do all objects (on Earth) fall

at the same rate?

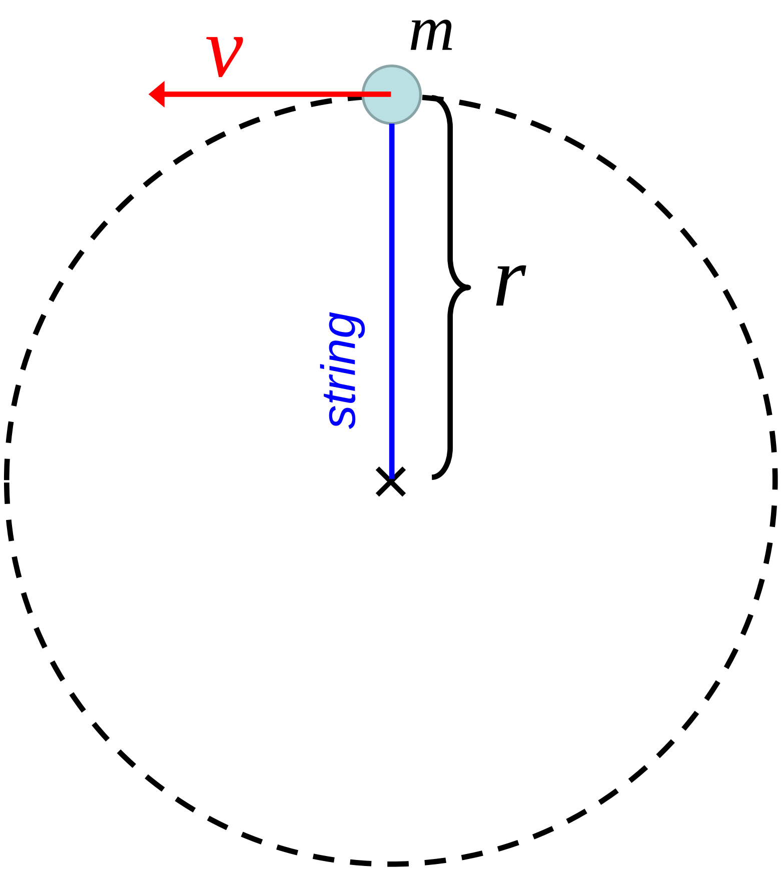
Circular Motion

Recall

acceleration = **change** in velocity over time

speed & direction

Rotation is a type of acceleration where the velocity **direction changes**, but speed is constant.



"ball on a string"

Circular Motion

<u>Recall</u>

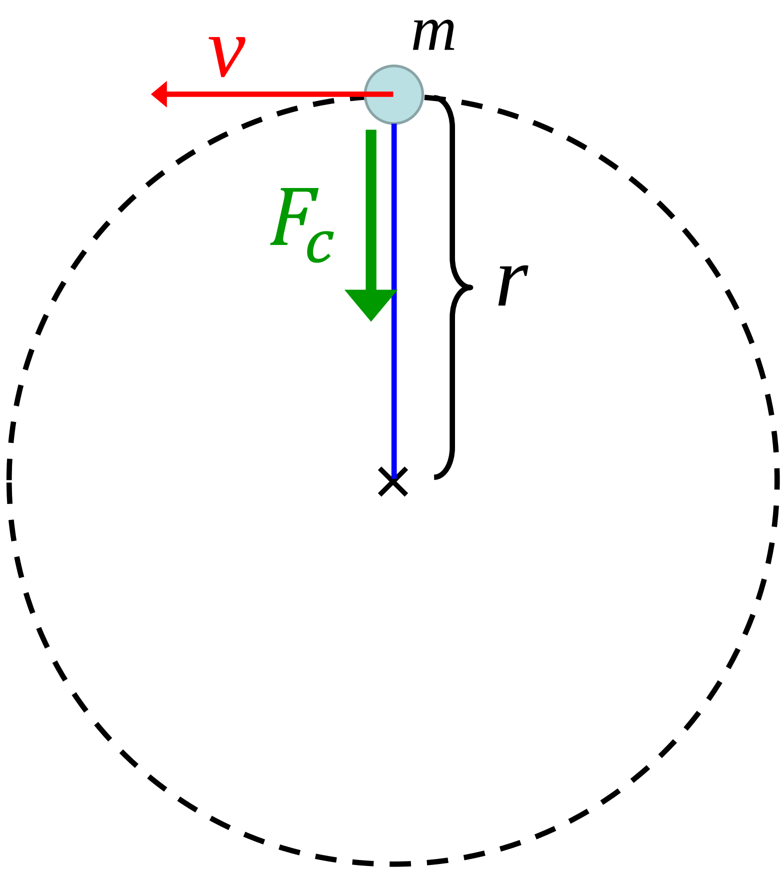
acceleration = **change** in velocity over time

speed & direction

Rotation is a type of <u>acceleration</u> where the velocity **direction changes**, but speed is constant.

Acceleration: $a_c = \frac{v^2}{r}$

Centripetal Force: $F_c = \frac{mv^2}{r}$

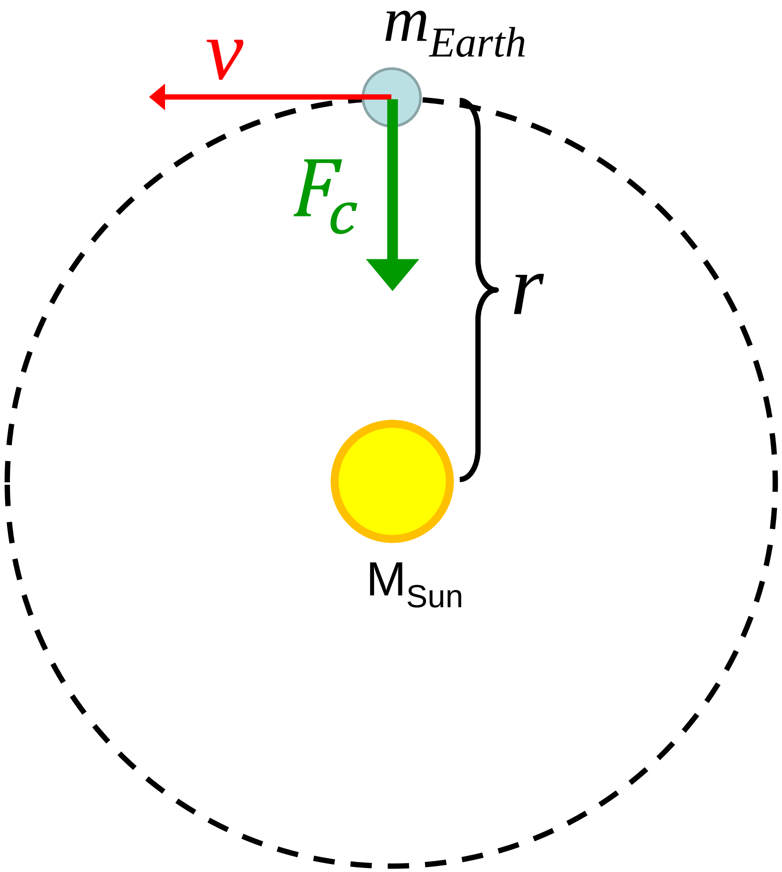


"ball on a string"

Circular Motion Example: Earth's orbit of Sun

Centripetal force needed to keep Earth on a circular orbit:

$$F_c = \frac{m_{Earth} v_{Earth}^2}{r}$$



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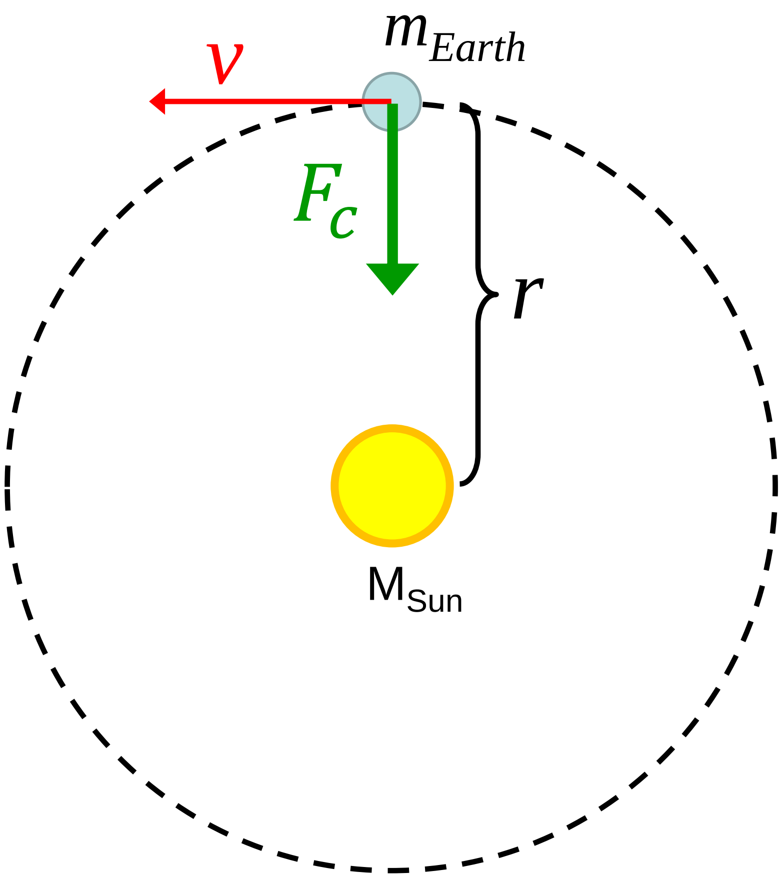

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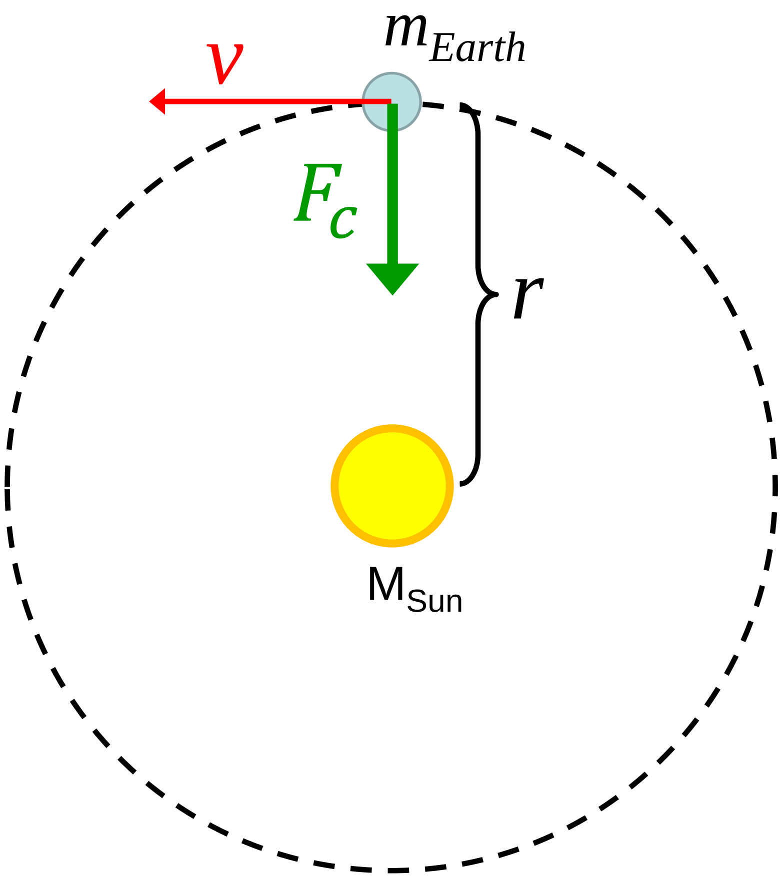

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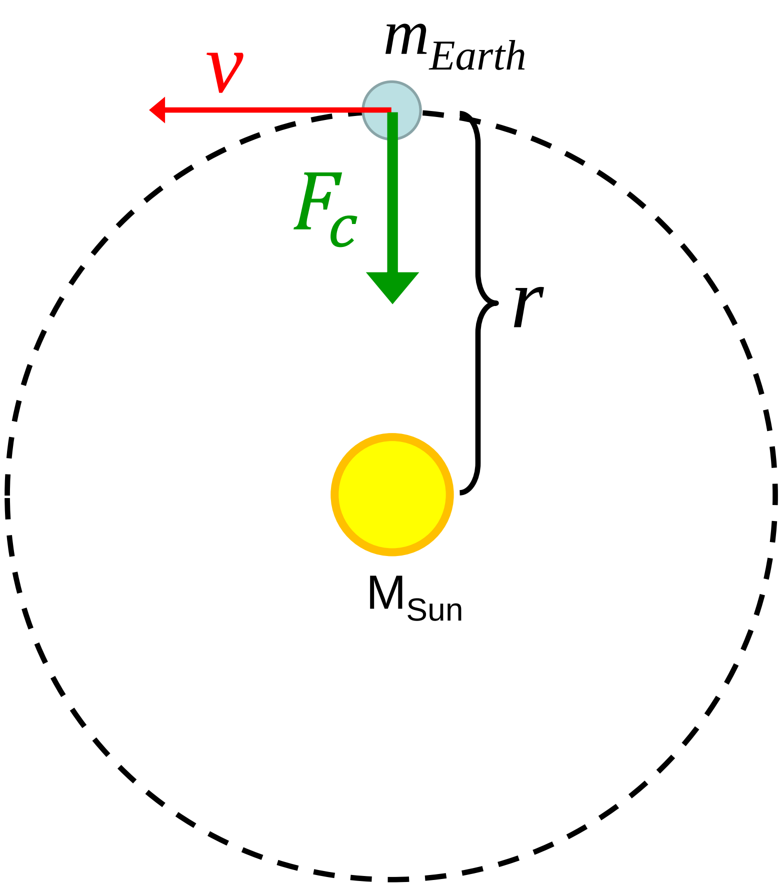

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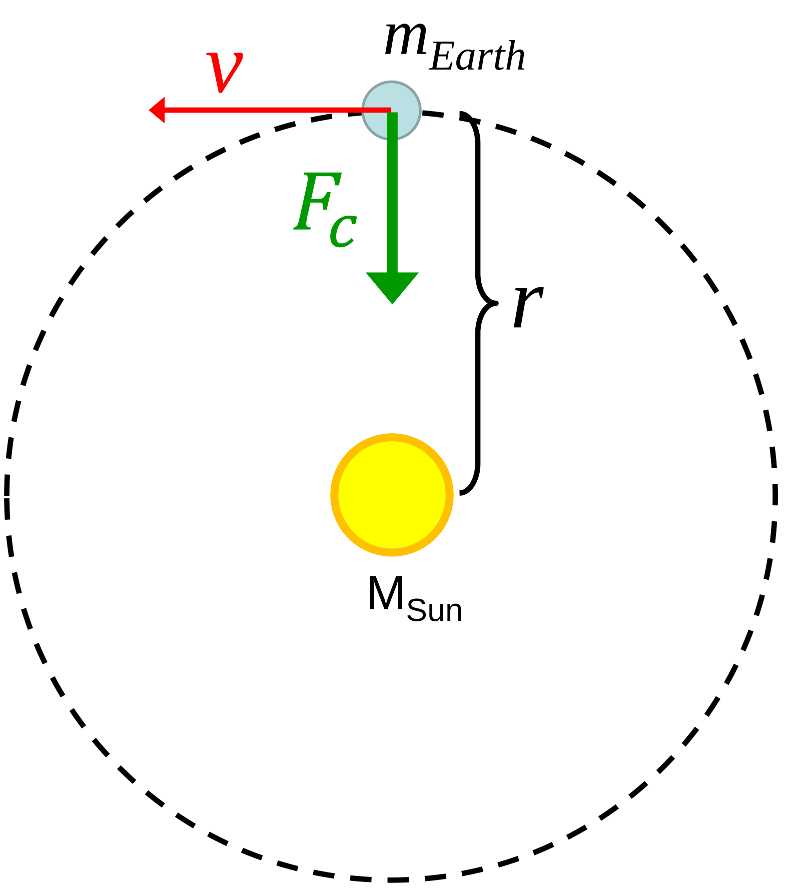

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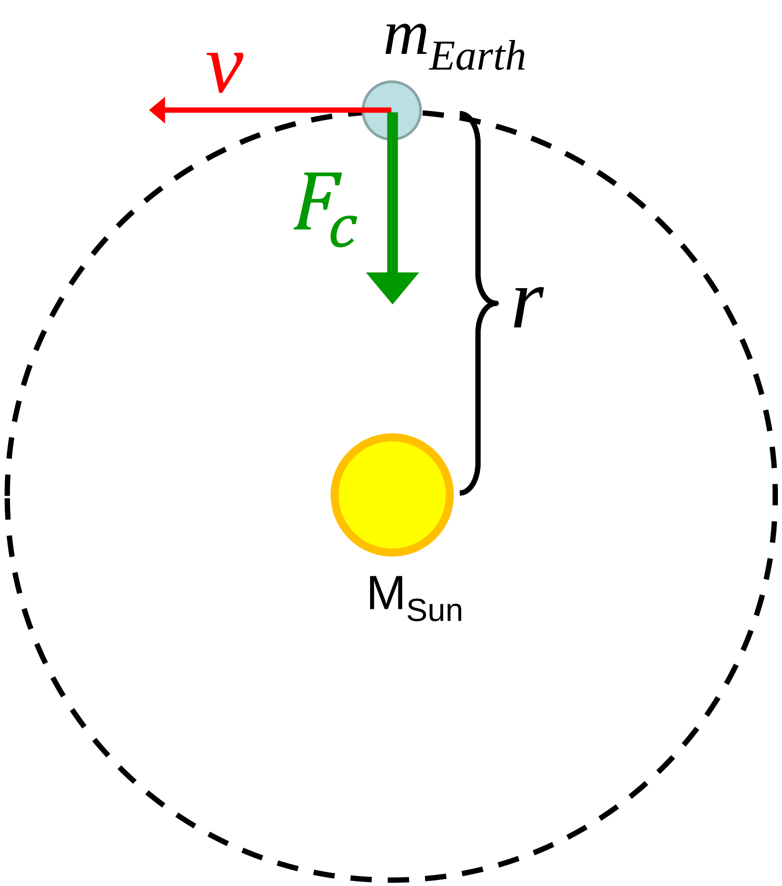
Solve for M_{Sun} : $$M_{Sun} = \frac{r\, v_{Earth}^2}{G}$$



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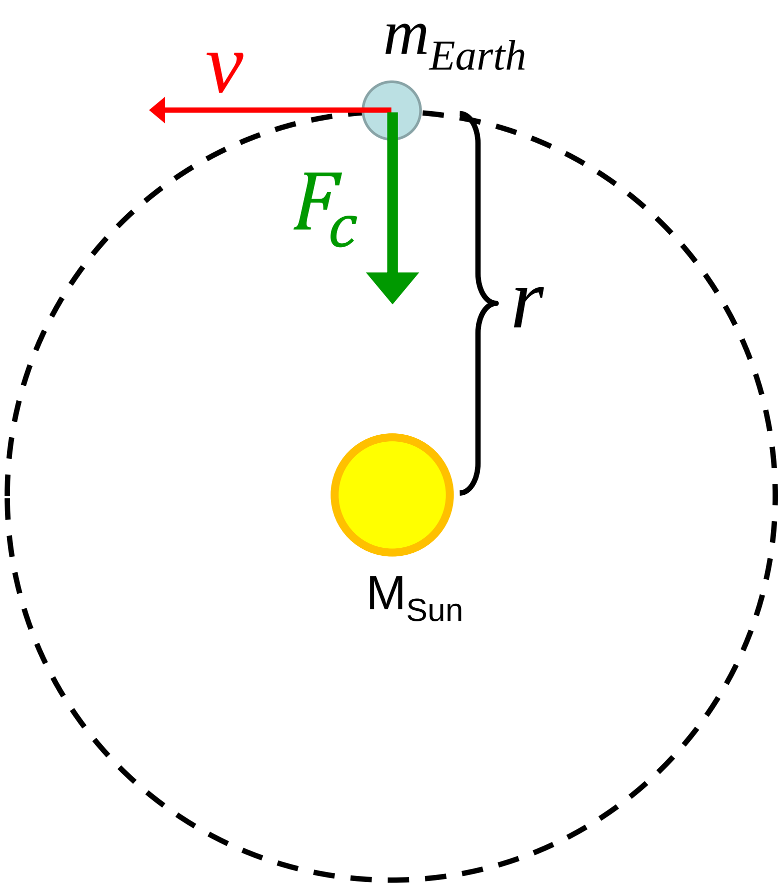
Solve for M_{Sun} : $$M_{Sun} = \frac{r\, v_{Earth}^2}{G}$$

$v_{Earth} = 29.78 \times 10^3$ m/s

$r = 1\,AU = 149.6 \times 10^9$ m

$G = 6.67430(15) \times 10^{-11}\, m^3/Kg \cdot s^2$



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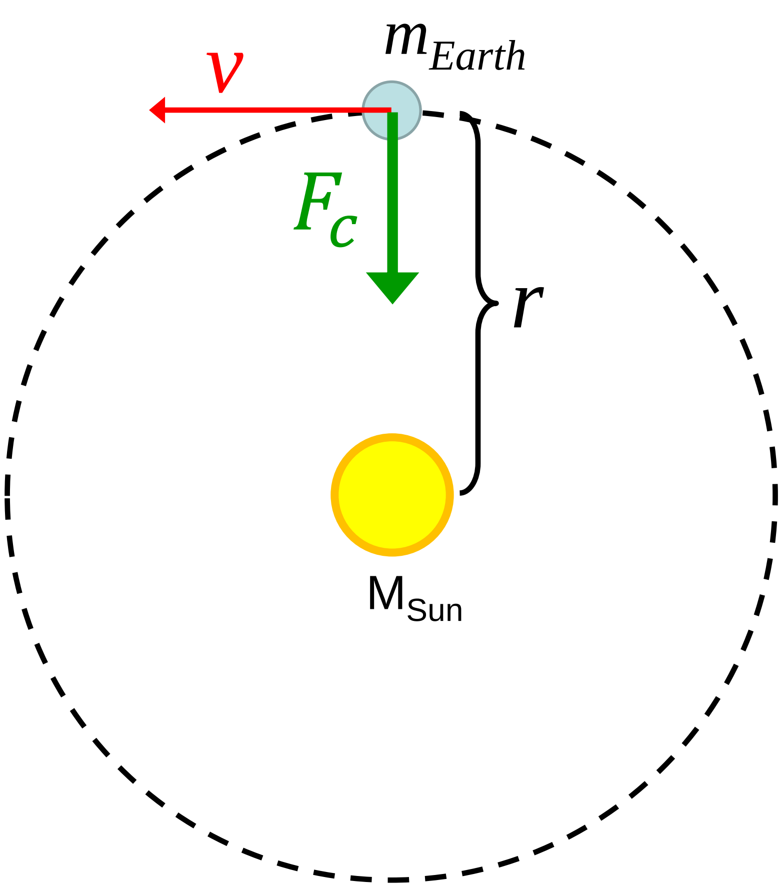
$$M_{Sun} = 1.988 \times 10^{30} \text{ Kg}$$



You can get the mass of the Sun from Earth's orbital parameters !!!

Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

This formula is in SI units

T = orbital period in seconds

a = semimajor axis in meters

$M_{1,2}$ =Mass of orbiting objects in Kg

G = 6.6743×10^{-11} $m^3/Kg.s^2$

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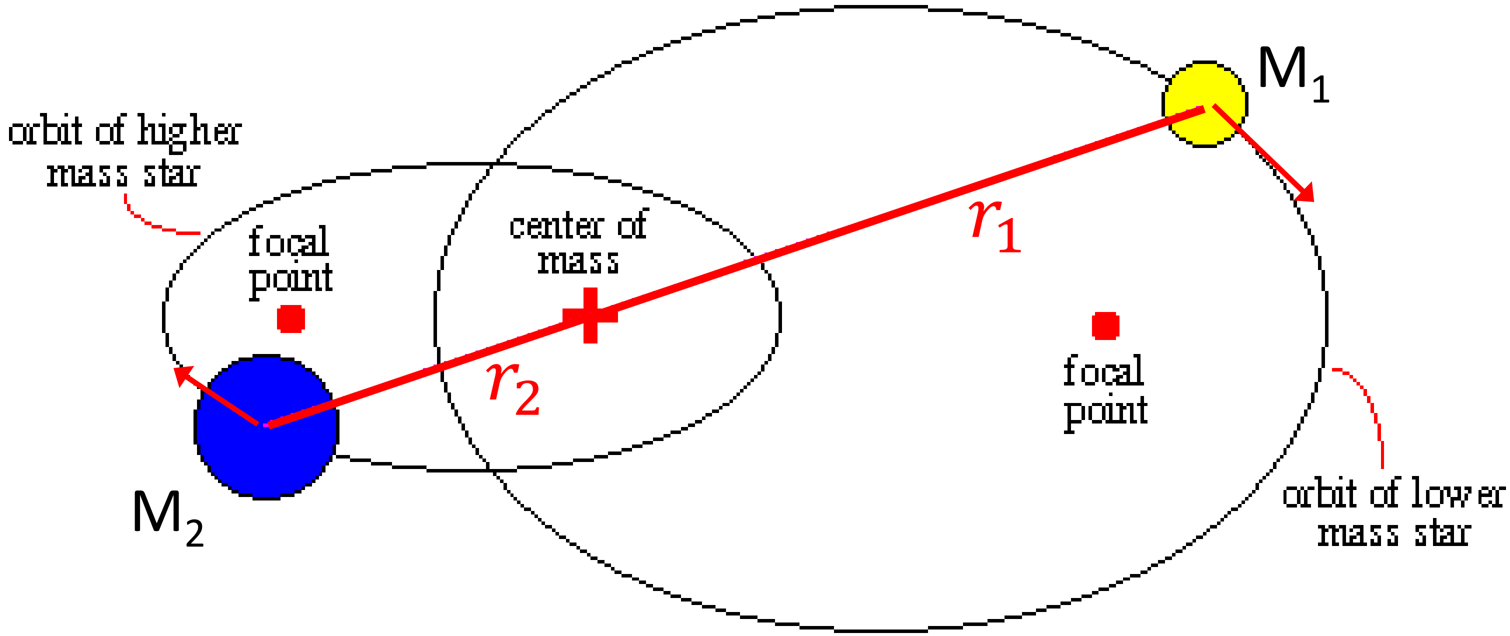
WHAT IF: What happens to the orbits if M_1 and M_2 are comparable ?

Center of Mass

What happens when $M_1 \simeq M_2$?

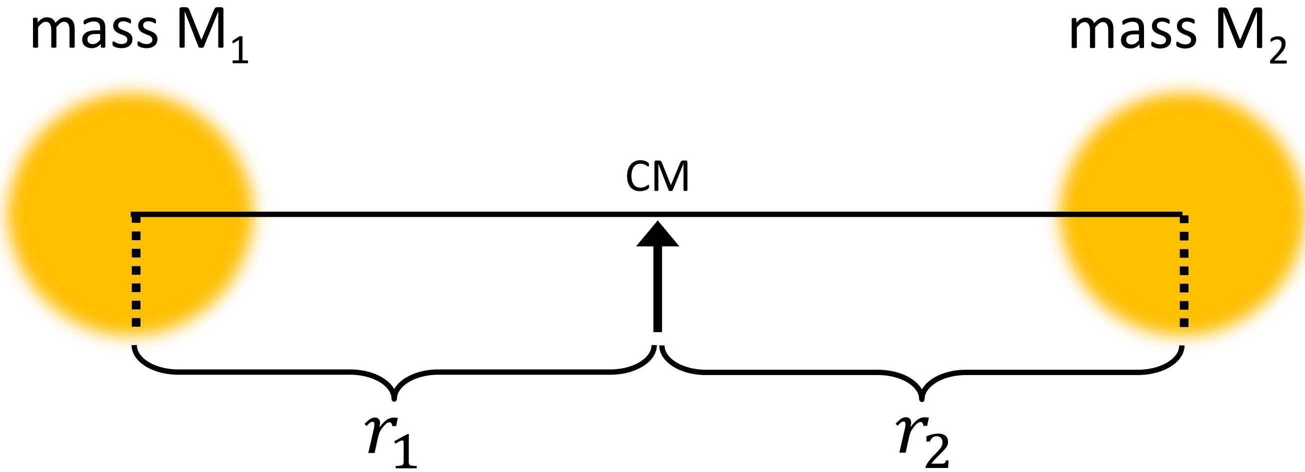
The **center of mass** of M_1 and M_2 serves as the orbiting ellipse focus.



[adapted from http://abyss.uoregon.edu]

Semimajor axis "a":
The coordinate "$r = r_1 + r_2$" is the distance between the two masses. It also describes an ellipse (not shown), whose semimajor axis "a" is used in Newton's version of Kepler's 3rd law.

Center of Mass



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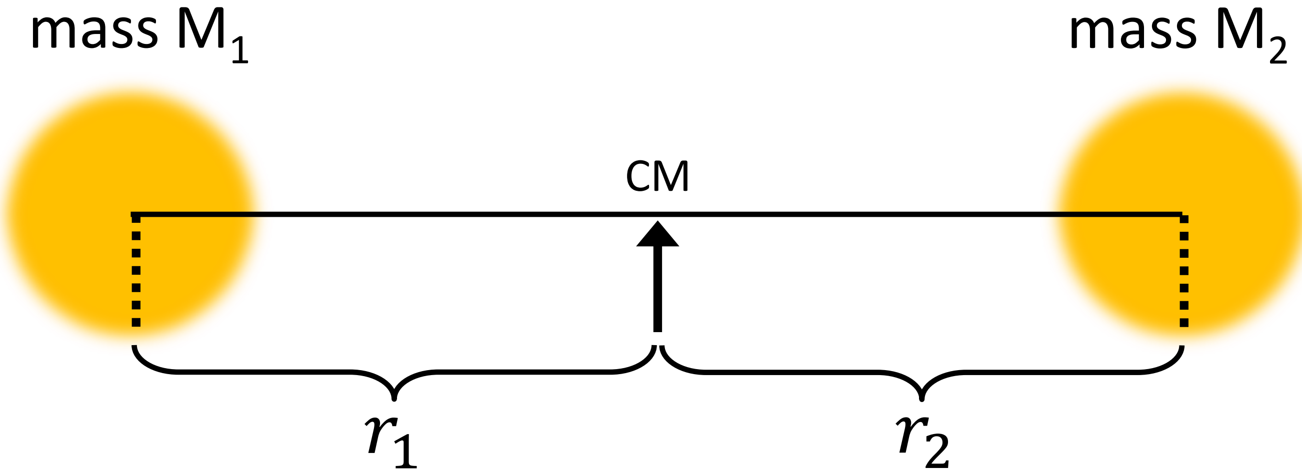

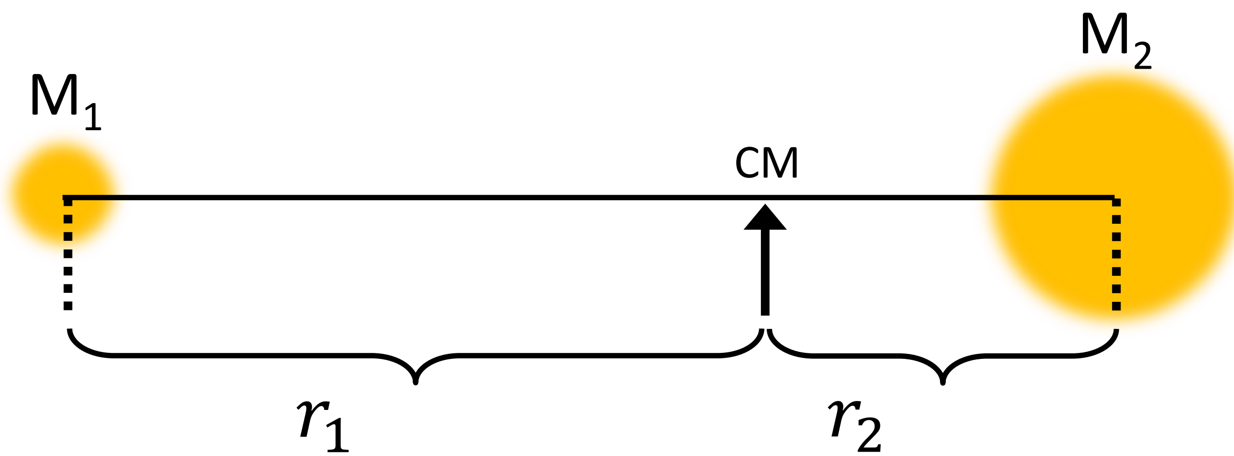

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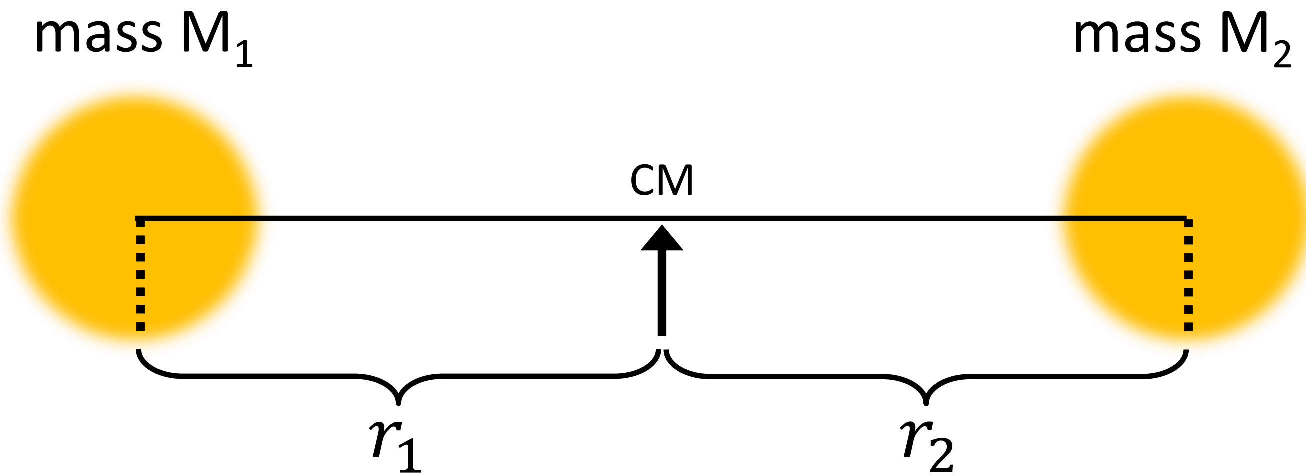
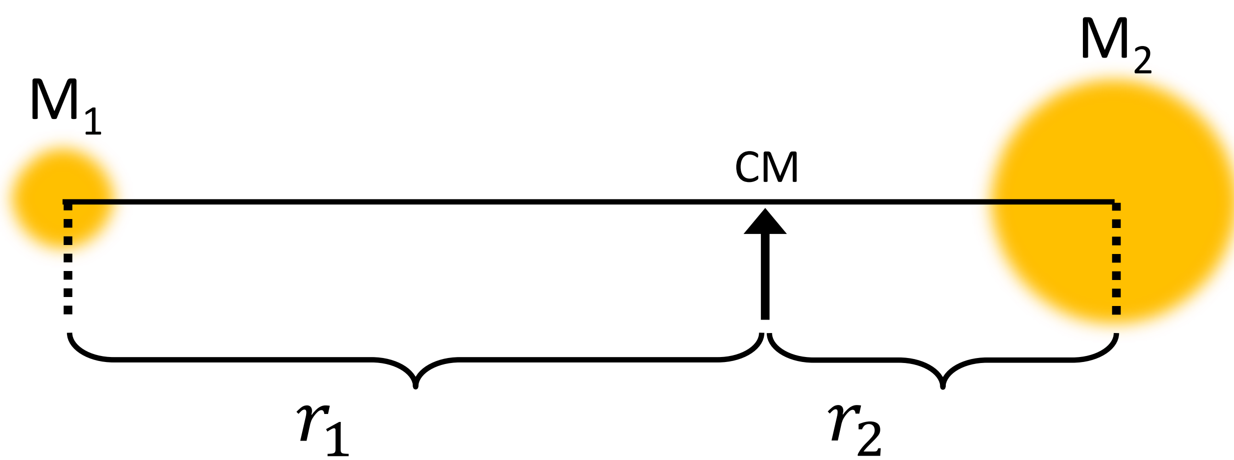


Center of mass is located such that $M_1 r_1 = M_2 r_2$

(or "barycenter")



Some Barycenters

M_2 - M_1: $r_2 = a\frac{M_1}{M_1+M_2}$ = distance from CM to M_2

Sun-Earth: $r_2 = 448\ km = 3.0 \times 10^{-6}$ AU

Earth-Moon: $r_2 = 4{,}670$ km with $a = 384{,}000$ km

= 73% of Earth's radius

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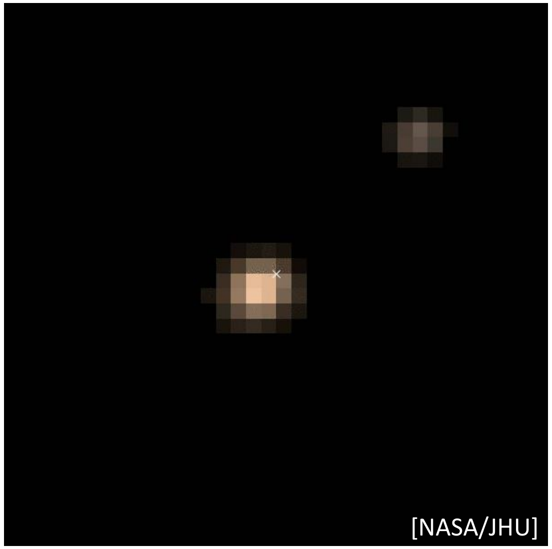
Pluto – Charon:

orbital period T = 6.4 days

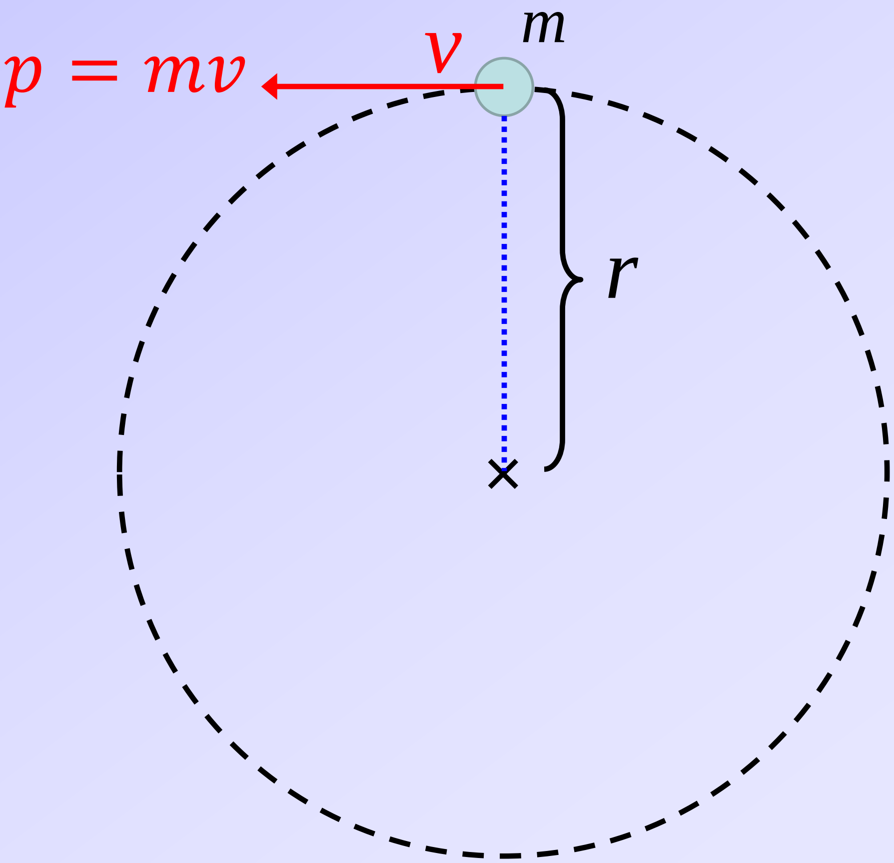


PollEv Quiz: PollEv.com/sethaubin

Conservation of Angular Momentum (1)

angular momentum $= L =$ momentum $\times$ radius

$= p \times r$ **...** $= mvr$ for circular motion



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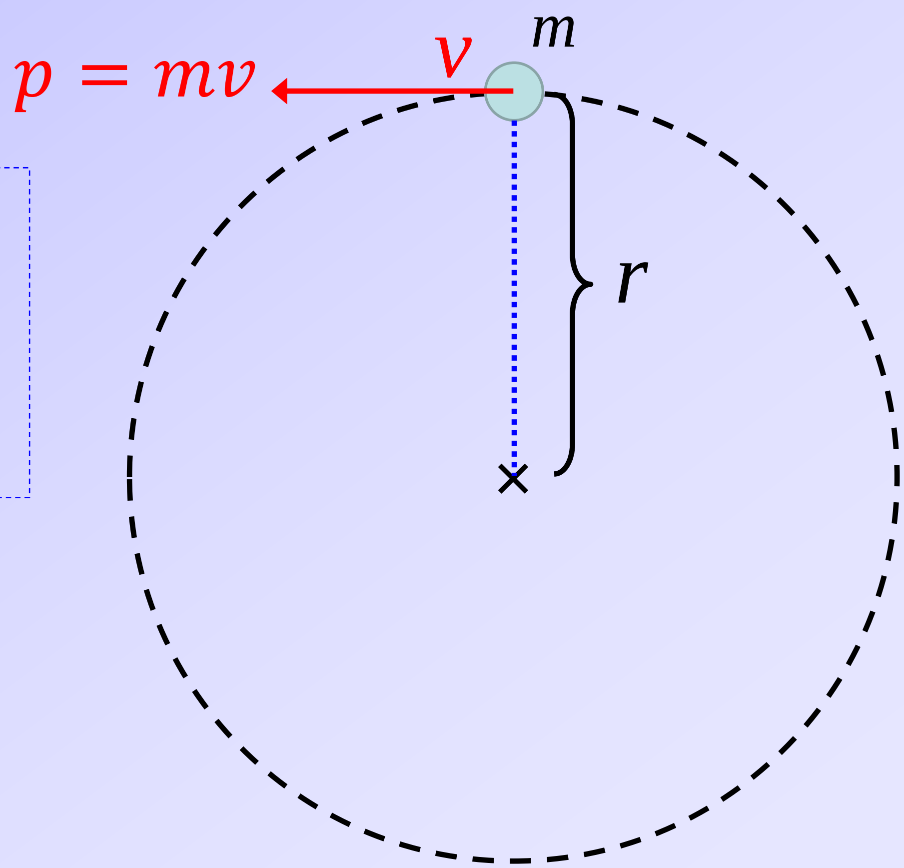

total angular momentum

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sum of the angular momenta of all the sub-parts of a system

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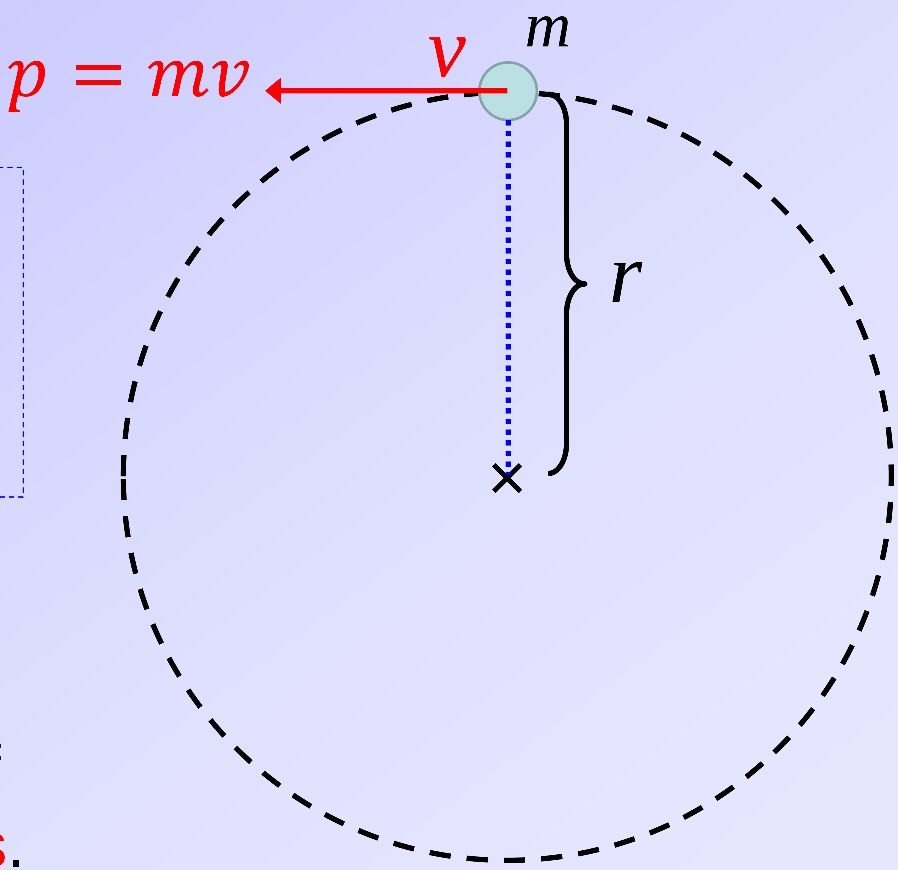

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Conservation Law

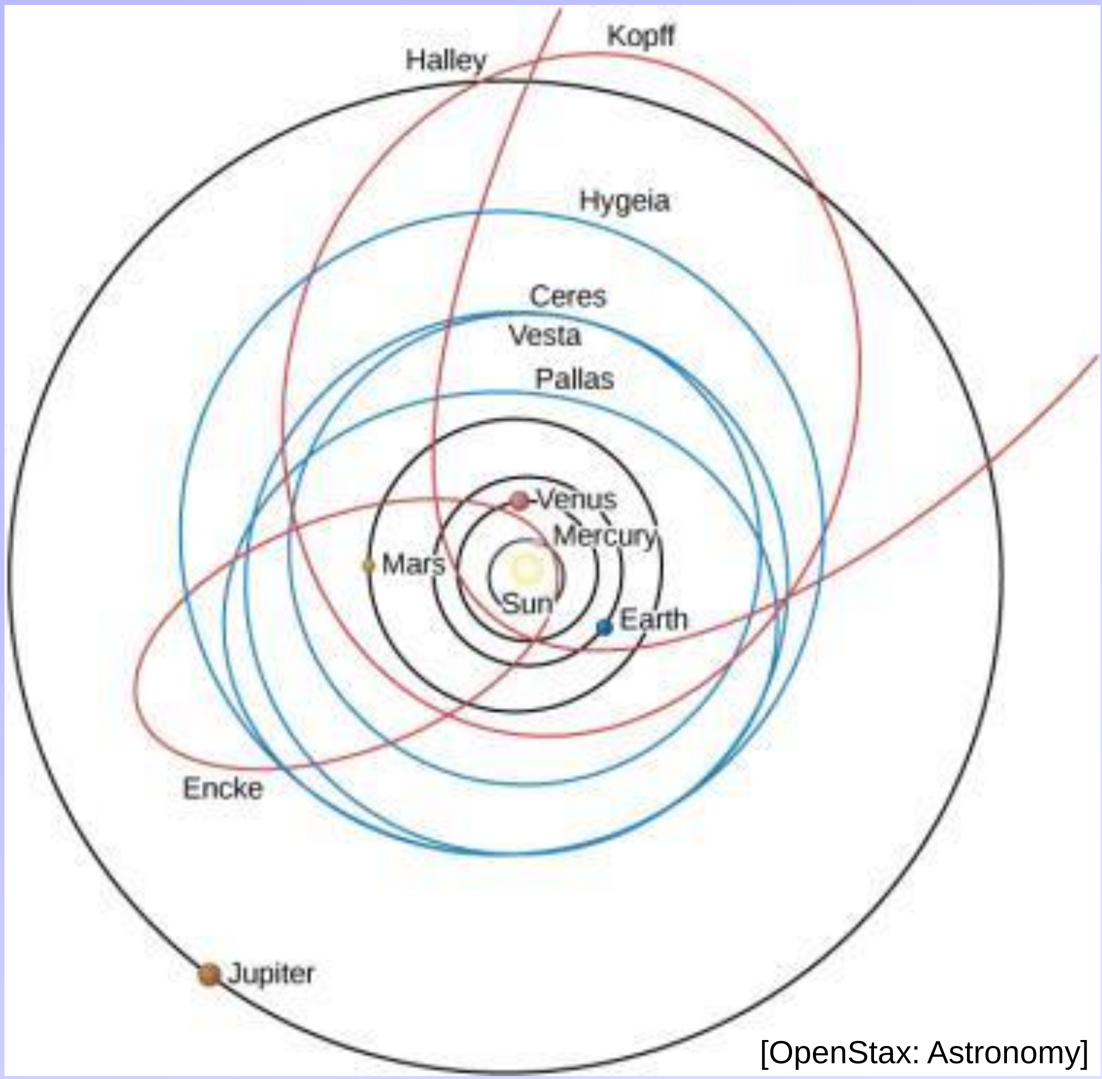
The total angular momentum of a closed system never changes.

Conservation of Angular Momentum (2)

[OpenStax: Astronomy]

- When a spinning figure skater **brings in her arms**, their distance from her spin center is **smaller**, so her speed increases.
- When her **arms are out**, their distance from the spin center is **greater**, so she **slows down**.

Conservation of Angular Momentum



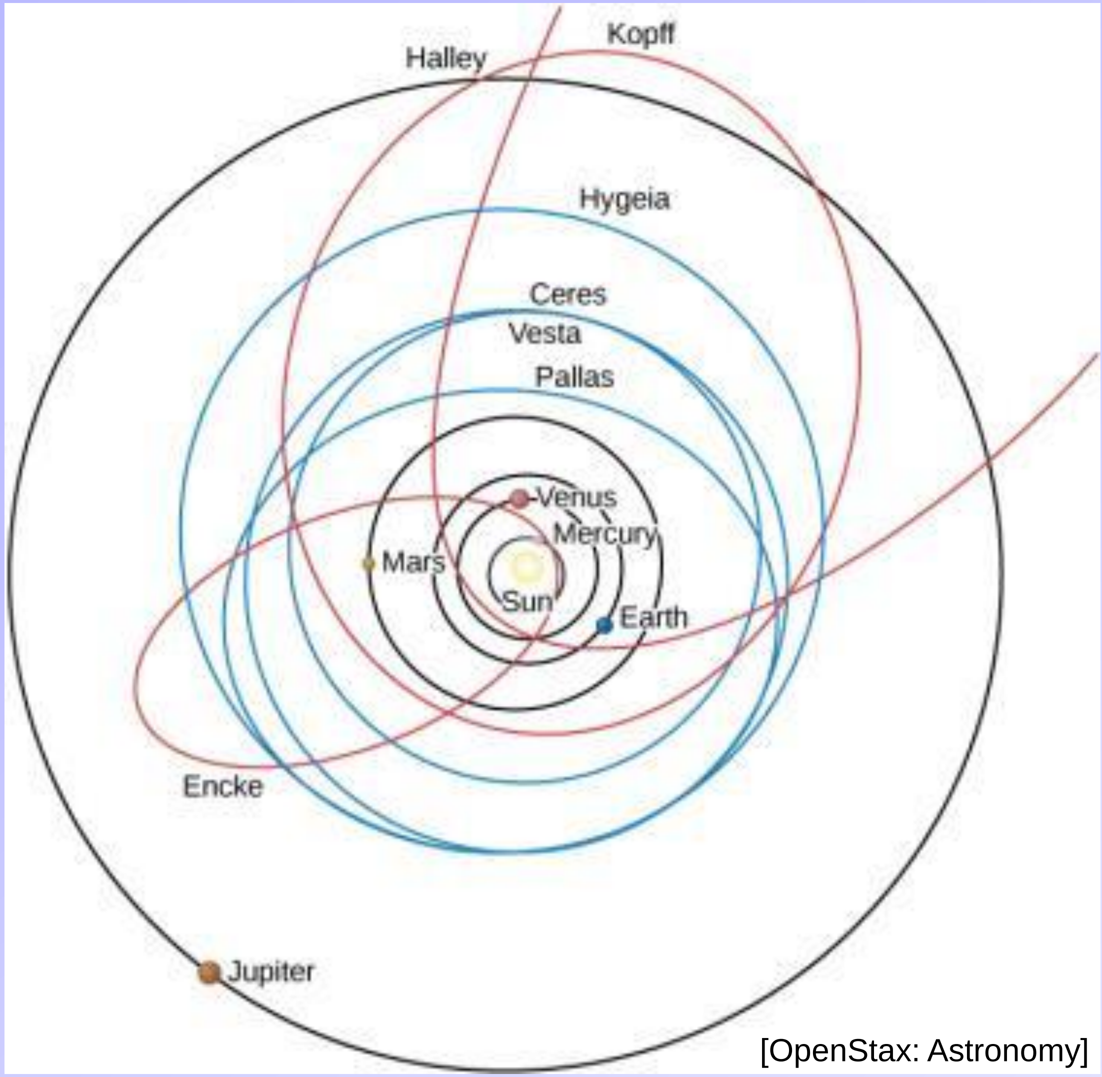
Planets (black), asteroids (blue), comets (red)

The multiple planets, asteroids, and comets all interact and modify each others orbits.

→ **Individual angular momenta change.**

→ **Total angular momentum of Solar System is constant.**

Conservation of Angular Momentum



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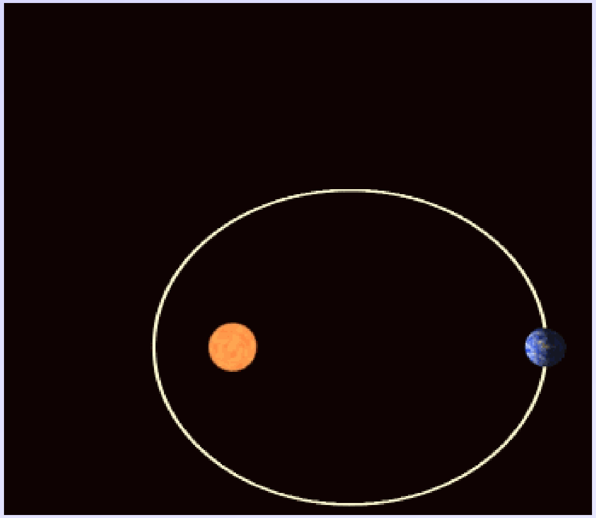
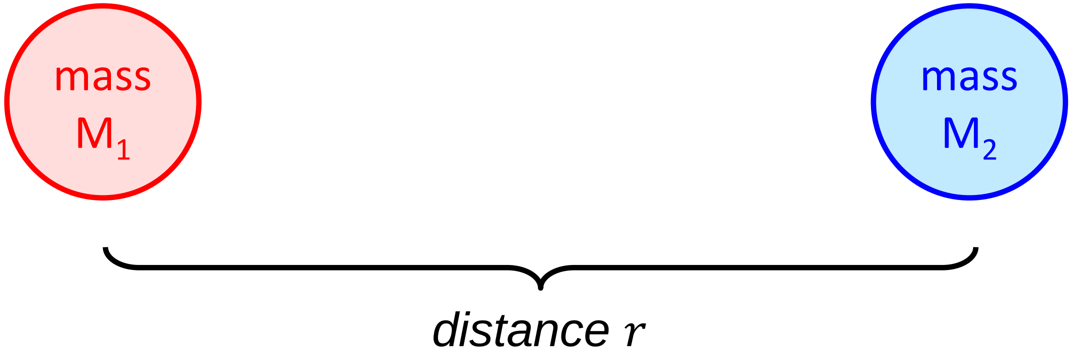
Example: Apsidal Precession



Gravitational Potential Energy



Stored gravitational energy $= E_{potential} = -\mathrm{G}\,\frac{M_1 M_2}{r}$

Total Energy $= E_{total} = E_{potential} + E_{kinetic}$

For 2 orbiting bodies (e.g. Sun + Earth): $E_{total} < 0$

For 2 unbound bodies (Earth + Mars rocket): $E_{total} > 0$

Bound Orbital Energy

Centripetal force needed to keep Earth on a circular orbit:

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Force of gravity on Earth from Sun:

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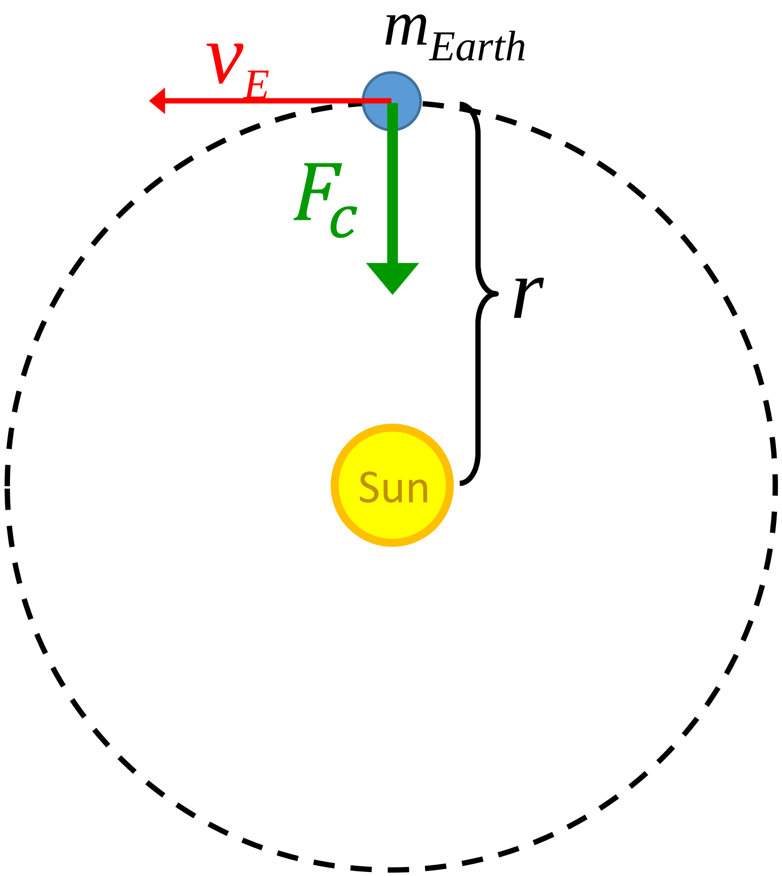

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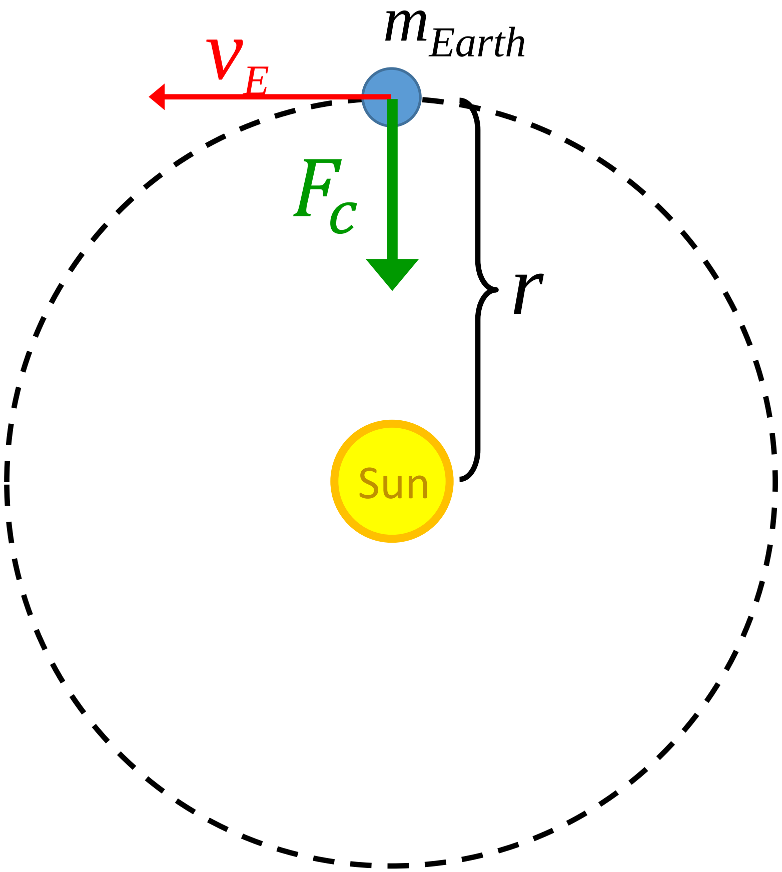

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$$\Leftrightarrow \quad \frac{m_{Earth} v_{Earth}^2}{r} = G\frac{m_{Earth} M_{Sun}}{r^2}$$

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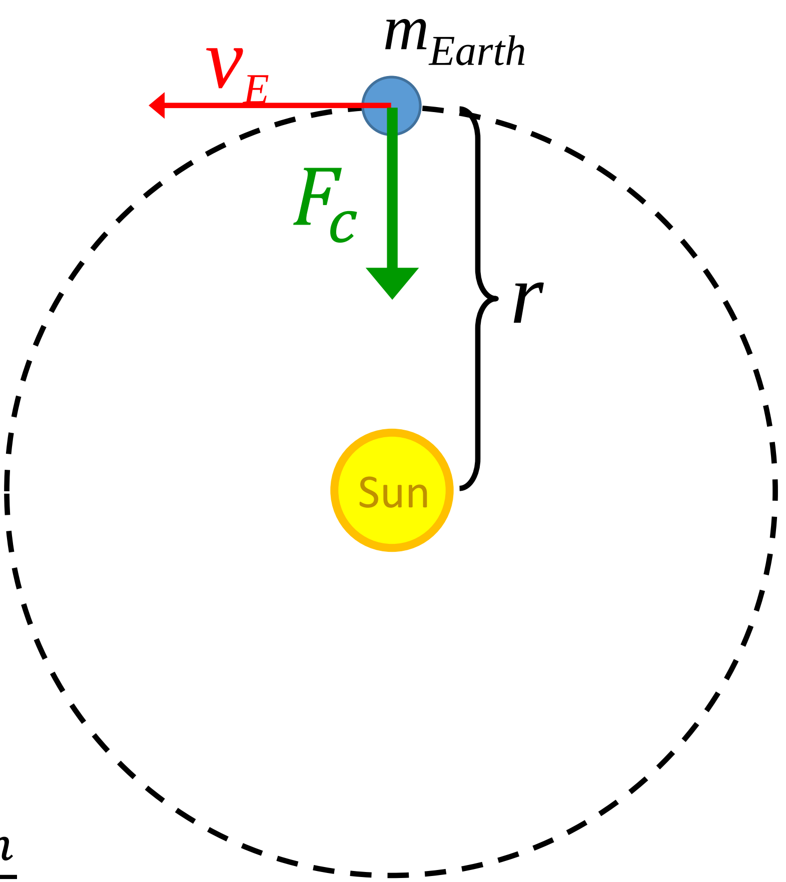

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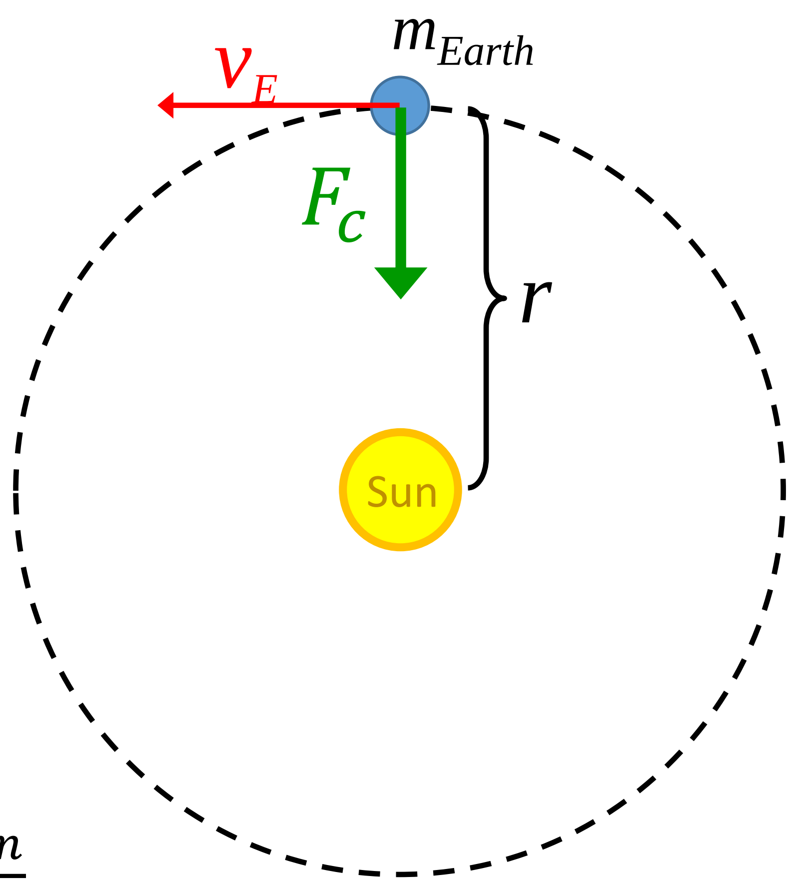
$$\Leftrightarrow \quad \frac{1}{2} m_{Earth} v_{Earth}^2 = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

$$\Leftrightarrow \quad E_{kinetic} = \frac{1}{2} m_{Earth} v_{Earth}^2 = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$



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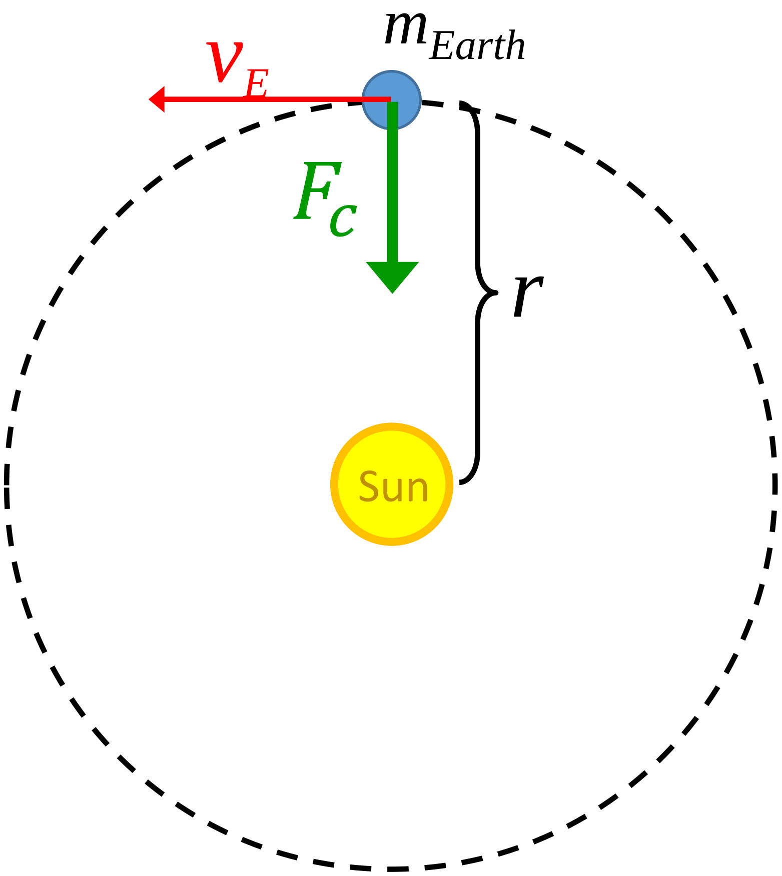
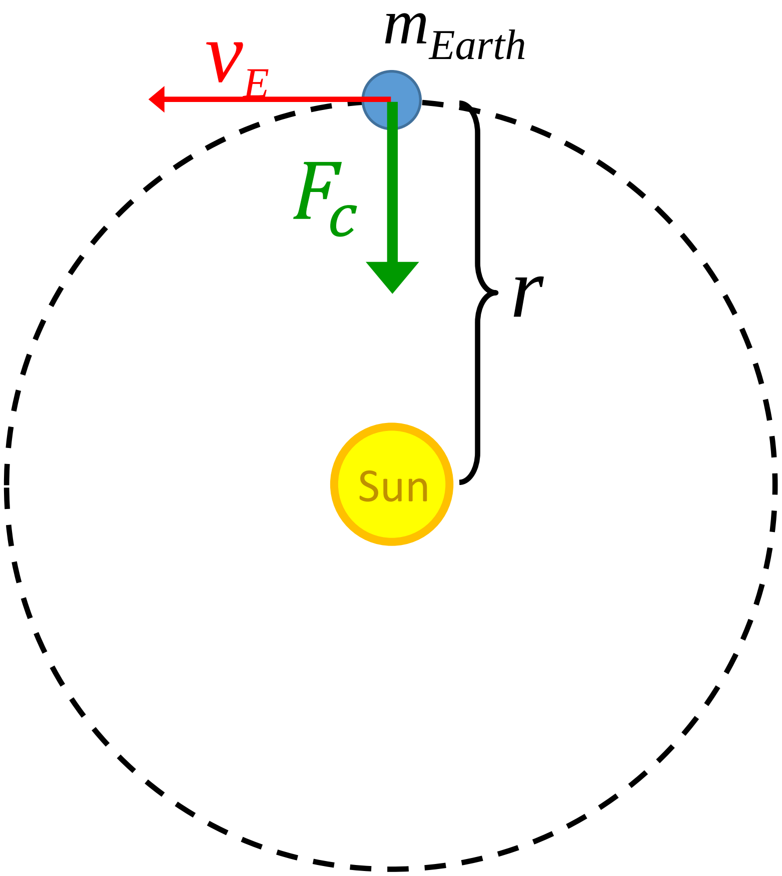


Bound Orbital Energy

$$E_{kinetic} = \frac{1}{2} G \frac{m_{Earth} M_{Sun}}{r}$$

Total Energy:

$$E_{Total} = E_{kinetic} + E_{potential}$$



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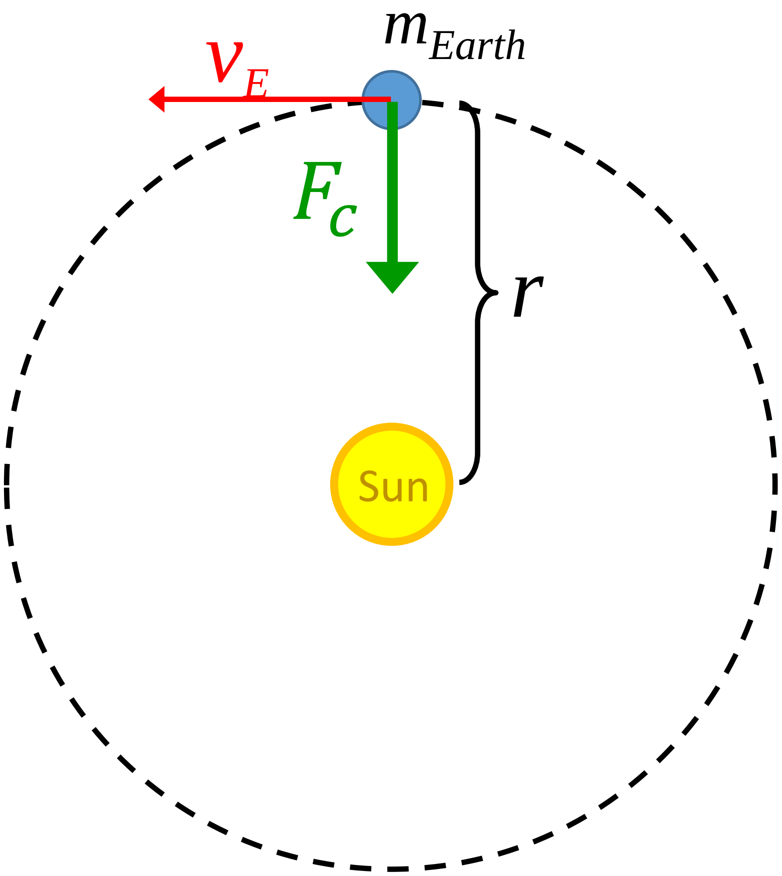

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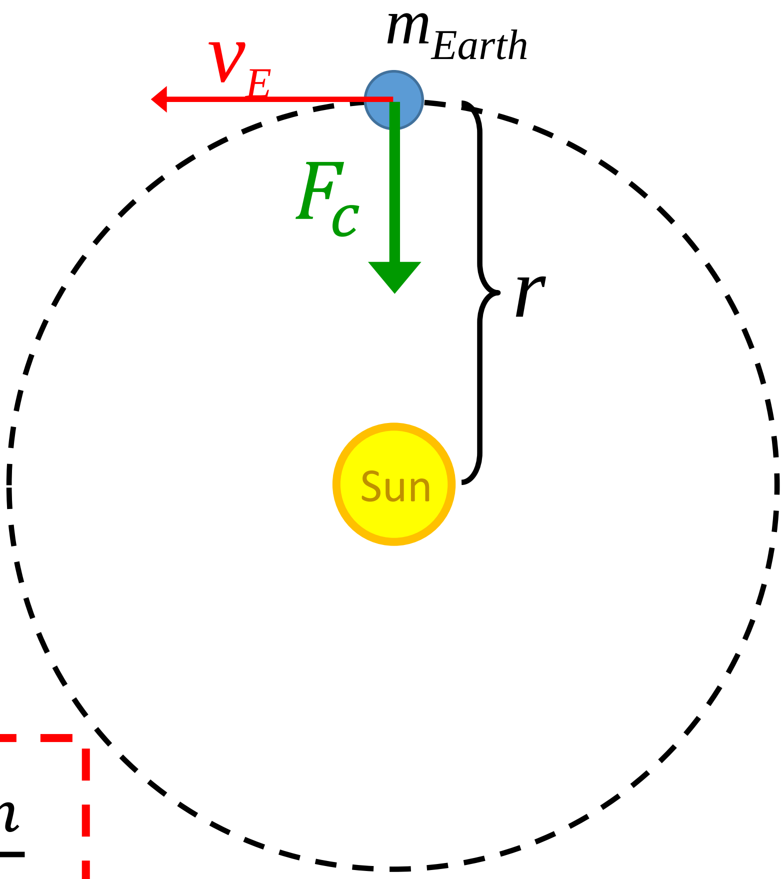

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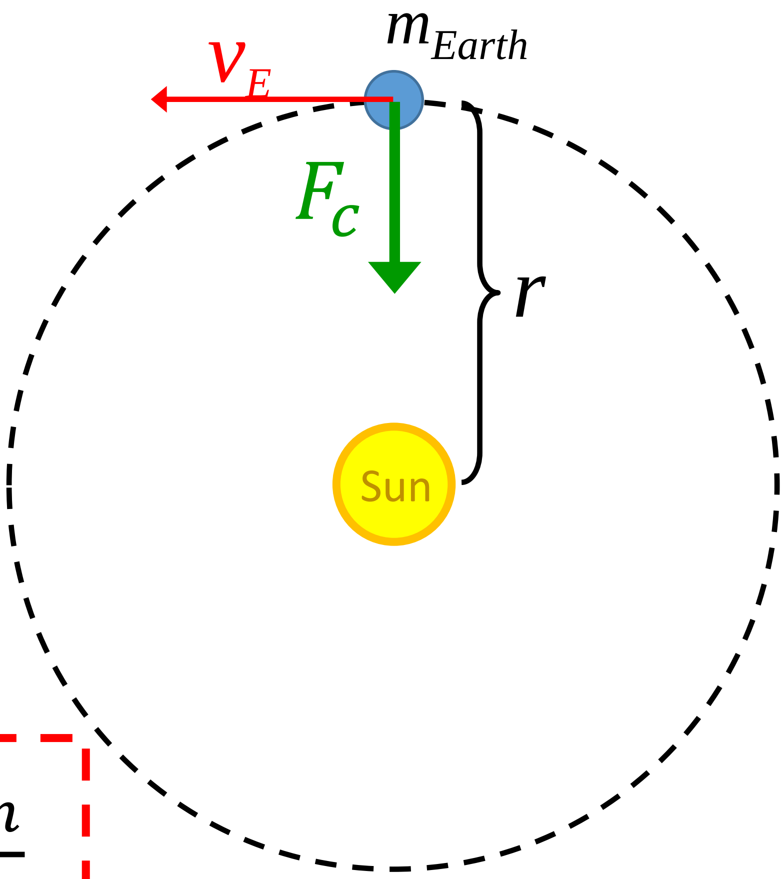

The bound orbital energy is negative: $E_{Total} < 0$

Example: *When a rocket wants to orbit another planet it has to slow (lower its energy) in order to go into orbit.*