Monday, February 10, 2025

Example: Newton's version of Kepler's 3rd Law for determining planetary mass.

If we assume that $M_{Moon} << M_{Earth}$, then we can measure M_{Earth} with the Moon's orbit.

Moon's orbital parameters

$$a = 384\,399 \text{ km} \simeq 3.84 \times 10^{8} \text{ m}$$

$$T = 27.3 \text{ days} \quad \text{(Moon's orbital period)}$$

$$= 27.3 \times 24 \times 60 \times 60 = 2358720 \simeq 2.36 \times 10^{6} \text{ s}$$

Newton's version of Kepler's 3rd Law:

$$T^2 = \frac{4\pi^2}{G\underbrace{(M_{Earth} + M_{Moon})}_{\simeq M_{Earth}}} a^3 \Rightarrow T^2 = \frac{4\pi^2}{G M_{Earth}} a^3$$

$$\Rightarrow M_{Earth} \simeq \frac{4\pi^2}{G} \frac{a^3}{T^2} = \frac{4(3.1415926)^2}{(6.6743 \times 10^{-11})} \frac{(3.84 \times 10^{8})^3}{(2.36 \times 10^{6})^2}$$

$$= 6.01 \times 10^{24} \text{ kg}$$

$$\Rightarrow \boxed{M_{Earth} \simeq 6.01 \times 10^{24} \text{ kg}}$$

Real Mass of Earth: $M_{Earth} = 5.97 \times 10^{24}$ kg