Wednesday, March 26, 2025

Example: Rotation frequency of a Sun-based white dwarf ... and $\underbrace{\text{"neutron star"}}$.

$\boxed{\text{major assumption: no mass loss}}$

the Sun cannot produce a neutron star, but this is a "what if" exercise.

note: $f_{sun} = \dfrac{1}{T_{sun}} = 4.63 \times 10^{-7}$ Hz

$T_{sun} = 25$ days $= 2.16 \times 10^{6}$ s

According to the Radius vs. mass plot, a 1 solar mass white dwarf has a radius of

$$R_{wd} = 0.0078\, R_{sun} = 5400 \text{ km}$$

For the neutron star, we will use $R_{NS} = 14$ km

$$= \frac{14}{6.96 \times 10^{5}} R_{sun}$$

$$\simeq 2 \times 10^{-5} R_{sun}$$

when the Sun contracts into a white dwarf (no mass loss) angular momentum is conserved.

$L = mvr$ for a particle going in a circle

$$v = \frac{2\pi r}{T} = 2\pi r f$$

(T: period; f: Frequency)

$\Rightarrow L = 2\pi r^2 f m$

"proportional"

for a rotating sphere: $L \propto 2\pi r^2 f m$

or $L = \alpha 2\pi r^2 f m$

proportionality constant that accounts for the geometry of the sphere

If angular momentum is conserved, then

$$\left.\begin{aligned} L_{sun} &= \alpha\, 2\pi\, r_{sun}^2\, f_{sun}\, m \\ L_{wd} &= \alpha\, 2\pi\, r_{wd}^2\, f_{wd}\, m \\ L_{ns} &= \alpha\, 2\pi\, r_{ns}^2\, f_{ns}\, m \end{aligned}\right\}$$

with $L_{sun} = L_{wd} = L_{ns}$

(m remains unchanged by assumption)

$$\text{Thus } \cancel{\frac{L_{sun}}{L_{wd}}}^{\,1} = \frac{\cancel{\alpha}\, \cancel{2\pi}\, r_{sun}^2\, f_{sun}\, \cancel{m}}{\cancel{\alpha}\, \cancel{2\pi}\, r_{wd}^2\, f_{wd}\, \cancel{m}} \Leftrightarrow \boxed{f_{wd} = \left(\frac{r_{sun}}{r_{wd}}\right)^2 f_{sun}}$$

$$\Rightarrow f_{wd} = \left(\frac{1}{0.0078}\right)^2 f_{sun} = (16437)(4.63 \times 10^{-7}) = 0.00761\ Hz$$

$$\boxed{\begin{aligned} &\Rightarrow f_{wd} = 0.0076\ Hz \\ &\Leftrightarrow T_{wd} = \frac{1}{f_{wd}} = 131\ s = 2 \text{ minutes } 11 \text{ seconds} \end{aligned}}$$

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neutron star: $\boxed{f_{ns} = \left(\frac{r_{sun}}{r_{ns}}\right)^2 f_{sun}} = \left(\frac{1}{2\times10^{-5}}\right)^2 f_{sun}$

$$= (2.5\times10^{9})(4.63\times10^{-7})$$

$$= 1153\ Hz$$

$$\Rightarrow \boxed{f_{ns} = 1153\ Hz \iff T_{ns} = 0.86\ ms = 0.86\times10^{-3}\ s}$$

neutron star rotation rate

neutron star rotation period