PHYS 172: Stellar Astronomy & Cosmology
Optional problems

Extra Practice Problems (ungraded)

1. Solar consumption

a) Calculate the equivalent mass loss of the Sun (due to emitted power) over the course of 1 second (kg/s) and over the course of 1 year (kg/yr). The output power of the Sun is 3.9×10^{26} W.

b) Assuming that the fusion only happens within the core of the Sun ($R_{core} = 0.2R_{Sun}$), calculate the average power generated per unit volume for solar fusion (answer in W/m^3).

c) The Sun is expected to "burn" about 10% of its total mass during its primary evolution (i.e. %10 of the mass participates in a fusion). Of this 10%, only a small fraction is actually converted to energy (you will have to look this up in the notes). Assuming that the Sun started with a mass of $\mathbf{M_{Sun}} = 1.99 \times 10^{30}$ kg, calculate the lifetime of the Sun during its primary evolution.

2. Binary star masses

Sirius is actually a binary star system: Sirius A (the "dog star" with mass M_A) is the bright star you see in the sky, and Sirius B ("the pup" with mass M_B) is a very faint white dwarf star. These two stars orbit each other, and, in this problem, you will determine the masses of these two stars from observational parameters of their orbital motion.

a) Total mass: The two stars orbit each other with a period of T = 50.1 years and an average distance from each other of a = 19.8 AU (i.e. semimajor axis). Calculate the total mass of the Sirius binary star system $M_{total} = M_A + M_B$ in kilograms.
Hint: Consider one of the versions of Kepler's laws.

b) Mass ratio: Telescope observations indicate that the center-of-mass of the star system is located 33% of the way from Sirius A on the line segment that connects the two stars. In other words, if we call a_1 the distance of M_A from the center-of-mass and a_2 the distance of M_B from the center-of-mass, then $a_1 = 0.33a$ and $a_2 = 0.67a$, with $a_1 + a_2 = a$.
Which star is heavier? Sirius A or Sirius B? Calculate the mass ratio M_A/M_B.
Hint: Review Lecture 7 (part A), which discusses center-of-mass physics.

c) Individual masses: You know $M_A + M_B$ and M_A/M_B. Determine M_A and M_B in kilograms and in units of solar masses (i.e. M_{Sun}).

3. Core collapse in a type-II supernova

Break the core collapse process in the type-II supernova of a massive star (e.g. 15 solar masses) into at least three steps from collapse of the core to the formation of the neutron star at the very center. Explain each of the steps in words, supported by a diagram for each step. Your explanation should include the nature of the initial core of the parent star, the role of neutrinos, and the rebounding shockwave (and destruction of the parent star).

You should also indicate the major forms of energy (and particles) released during the explosion, as well as the types of heavy elements that are produced.

Note: You can use the book, the course notes (slides and lectures), and any other sources (e.g. Wikipedia) to help you with this problem.

4. The Crab Nebula neutron star

The neutron star (pulsar) at the center of the Crab Nebula has been estimated to have a mass of $M_{ns} = 1.4\ M_{Sun}$, a radius of $R_{ns} = 14$ km, and a surface temperature of $T_{ns} = 1.6 \times 10^6$ K. (disclaimer: in reality, there is a fair bit of uncertainty on these numbers, but we will use these for this problem). The neutron star rotates with a period of 33.5 milliseconds.

a) Calculate the acceleration due to gravity at the surface of the neutron star (in m/s^2).

b) Calculate the speed (in m/s) of an object on the equator of the neutron star (due its rotation). What fraction of the speed of light is the equatorial speed.

c) Calculate the centripetal acceleration (in m/s^2) of an object at the equator due to the rotation of the neutron star and compare it to the gravitational acceleration in part a).

d) Calculate the peak thermal emission wavelength (in nm) and indicate the region of the electromagnetic spectrum that is belongs to. Calculate the photon energy associated with wavelength in Joules and electron volts (i.e. eV).

e) Calculate the surface intensity (in W/m^2) of the blackbody radiation emitted by the neutron star. Use this result to calculate the thermal output power (in W) of the neutron star, i.e. luminosity, and convert it into units of the Sun's luminosity L_{Sun}.

5. Neutrino energy in a supernova

A type II supernova typically releases 10^{46} Joules of energy, of which 99% is emitted in the form of about 10^{58} neutrinos (you can approximate 99% to 100% in this problem). These neutrinos are all emitted in a roughly 10 s span at the start of the explosion (before any light is emitted).

a) Calculate the average energy carried by each neutrino in Joules and in MeV.

Note: since you are doing an order of magnitude calculation, you will not get the measured value of 10-15 MeV, though you should not be too far off.

b) How much mass is converted to neutrinos during the supernova explosion? Give your answer in kilograms and in units of solar mass, i.e. M_{Sun}.

c) As a crude approximation, we can consider the neutrinos emitted during the supernova as the result of "blackbody neutrino emission" (i.e. we will treat the neutrinos as pseudo-photons). Use your result of part a) to calculate the "frequency" (using $E = hf$) and "wavelength" of the neutrinos (using $\lambda f = c$). Convert your wavelength to nanometers and use Wien's law to estimate the temperature of the collapsed core that emits the "blackbody neutrinos."

Note: Your answer for the temperature will be an underestimate. Current models estimate the collapsed core to be 100 billion Kelvin.