

Friday; April 4, 2025

#1

Example: Time dilation near black hole.

Q: How much slower does a clock run when it is 1 m from the event horizon of a black hole?

i.e. How much faster does the rest of the universe tick?

Black hole at the center of the Milky Way (Sagittarius A*)

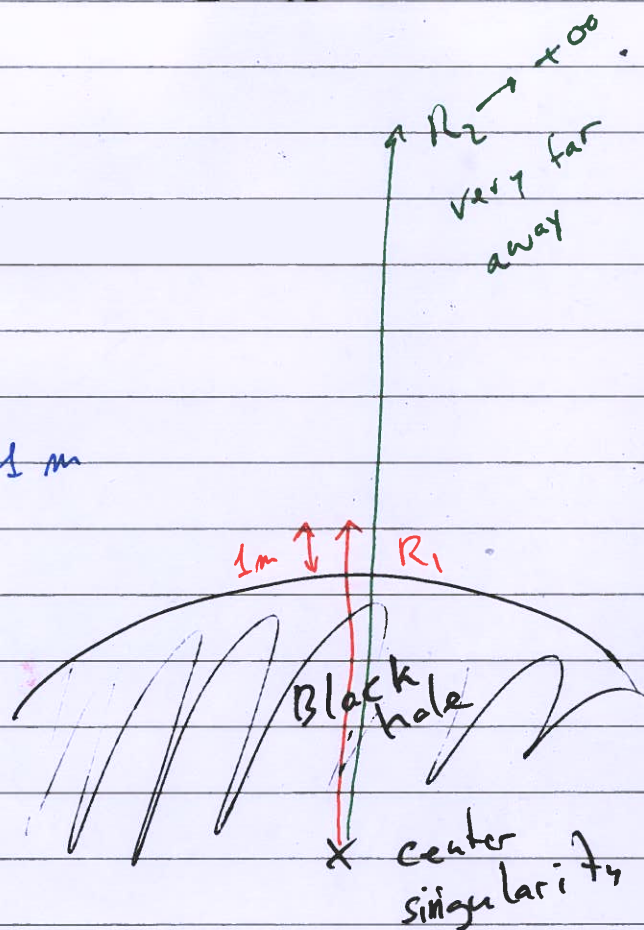
$$\begin{aligned} R_s &\approx 17 R_{\text{sun}} \\ &= 12 \times 10^6 \text{ km} \\ &= 12 \times 10^9 \text{ m} \end{aligned}$$

$$\frac{\text{clock 2 frequency}}{\text{clock 1 frequency}} = \frac{f_2}{f_1}$$

$$\frac{f_2}{f_1} = \frac{\sqrt{1 - \frac{R_s}{R_2}}}{\sqrt{1 - \frac{R_s}{R_1}}}$$

$R_1 \approx R_s + 1 \text{ m}$

$$= \frac{1}{\sqrt{1 - \frac{R_s}{R_1}}}$$



$$\begin{aligned}
 &= \sqrt{\frac{1}{\frac{R_1}{R_1} - \frac{R_s}{R_1}}} = \sqrt{\frac{1}{\frac{R_1 - R_s}{R_1}}} \\
 &= \sqrt{\frac{R_1}{R_1 - R_s}} = \sqrt{\frac{12 \times 10^9 + 1}{1}} \\
 &\quad \quad \quad = 1 \text{ m} \\
 &\quad \quad \quad \approx \sqrt{12 \times 10^9} \\
 &\quad \quad \quad = 1.095 \times 10^5 \\
 &\quad \quad \quad \approx 110,000
 \end{aligned}$$

At 1 m from the event horizon, the rest of the universe evolves at ~~at~~ a rate 110,000 times faster than you.

note: at 1 m from the event horizon of the M87 black hole, the rest universe evolves at a rate 4.4×10^6 times faster than you.

$$[R_{s_{\text{M87}}} = 128 \text{ AU} = 1.92 \times 10^{13} \text{ m}]$$