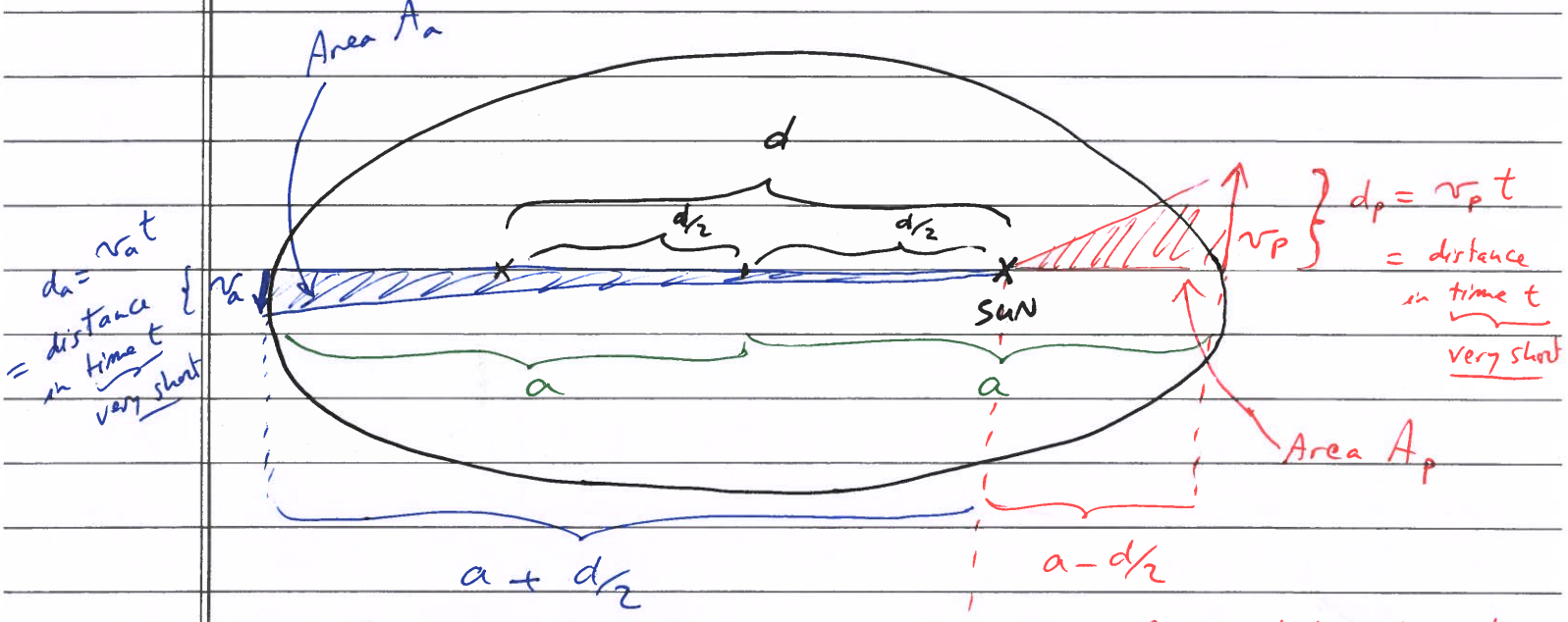
Friday, January 30, 2026

Speeds at aphelion and perihelion

(far, slow) (near, fast)

→ Determine $\frac{v_p}{v_a}$



Area of aphelion triangle

$$A_a = \frac{(a + d/2)(v_a t)}{2}$$

Area of perihelion triangle

$$A_p = \frac{(a - d/2)(v_p t)}{2}$$

Kepler's 2nd law: Area of aphelion triangle = Area of perihelion triangle

$$A_a = A_p$$

$$\frac{(a + \frac{d}{2})(v_a t)}{2} = \frac{(a - d/2)(v_p t)}{2}$$

$$\Leftrightarrow \quad \frac{v_p}{v_a} = \frac{a + d/2}{a - d/2} = \frac{2a + d}{2a - d}$$

multiply by $\frac{2}{2}$

divide numerator and denominator by "2a"

ε = eccentricity

$$\frac{v_p}{v_a} = \frac{\frac{2a}{2a} + \frac{d}{2a}}{\frac{2a}{2a} - \frac{d}{2a}}$$

$\frac{2a}{2a} \to 1$; $\frac{d}{2a} \to \varepsilon$

$$\Leftrightarrow \quad \boxed{\frac{v_p}{v_a} = \frac{1 + \varepsilon}{1 - \varepsilon}}$$

Example 1: Earth with $\varepsilon = 0.02$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.02}{1 - 0.02} = \frac{1.02}{0.98} \simeq 1.04$$

$\Rightarrow$ Earth's orbital speed varies by 4%

Example 2: Mars with $\varepsilon = 0.09$

$$\frac{v_p}{v_a} = \frac{v_{max}}{v_{min}} = \frac{1 + 0.09}{1 - 0.09} \simeq 1.2$$

$\Rightarrow$ Mars's orbital velocity varies by 20%