Monday, April 13, 2026

Example: Estimating the mass of the Milky Way inside the Sun's galactic orbit.

The solar system (i.e., Sun) travels on a roughly circular orbit with radius $R = 26$ kly, velocity $= 230 \frac{km}{s}$ and an orbital period $T \simeq 226$ million years.

Convert to SI units:

$$R = 26 \text{ kly} = 26 \times 10^3 \text{ light years} = 2.44 \times 10^{17} \text{ km} = 2.44 \times 10^{20} \text{ m}$$

($1 \text{ light year} \simeq 9.4 \times 10^{12}$ km)

$$\text{velocity} = v = 230 \text{ km/s} = 230 \times 10^3 \text{ m/s}$$

$$T = 226 \text{ million years} = 226 \times 10^6 \text{ years} = 7.12 \times 10^{15} \text{ s}$$

($1 \text{ year} \simeq \pi \times 10^7$ s)

Method 1: Newton's version of Kepler's 3rd Law

$$T^2 = \frac{4\pi^2}{G\left(M_{\text{galaxy inside}} + M_{\text{sun}}\right)} a^3$$

a = semi-major axis = radius for a circular orbit

neglect $M_{\text{galaxy}} \gg M_{\text{sun}}$

$$\Rightarrow T^2 = \frac{4\pi^2}{G M_{\text{galaxy inside}}} R^3 \iff M_{\text{galaxy inside}} = \frac{4\pi^2}{G} \frac{R^3}{T^2}$$

$$\Rightarrow M_{\substack{galaxy \\ inside}} = \frac{4(3.1415)^2}{6.6743\times10^{-11}} \frac{(2.44\times10^{20})^3}{(7.12\times10^{15})^2}$$

$$= 1.695\times10^{41}\ kg \quad \div M_{sun} = 1.99\times10^{30}\ kg$$

$$= 8.52\times10^{10}\ M_{sun}$$

$$= 85 \text{ billion } M_{sun}$$

Method 2 : Gravitational Force

Force of gravity: $F_G = \frac{G M_{galaxy} M_{sun}}{R^2}$

Centripetal Force for circular motion: $F_c = \frac{M_{sun} v^2}{R}$

$$F_G = F_c \iff \frac{G M_{galaxy} M_{sun}}{R^2} = \frac{M_{sun} v^2}{R}$$

$$\iff M_{galaxy} = \frac{R v^2}{G} = \frac{(2.44\times10^{20})(230\times10^3)^2}{6.6743\times10^{-11}}$$

$$= 1.934\times10^{41}\ kg \quad \div M_{sun} = 1.99\times10^{30}\ kg$$

$$= 9.72\times10^{10}\ M_{sun}$$

$$= 97 \text{ billion } M_{sun}$$