Wednesday, April 29, 2026

Example: Critical density of the universe to bring its expansion to a halt at infinite time. (scenario 3)

Einstein's equations of General Relativity lead to the following expression for the critical density:

$$\rho_{critical} = \frac{3H^2}{8\pi G}$$

Hubble's constant (H)

Newton's gravitational constant (G)

$$G = 6.6743 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$H = \frac{22\ km/s}{\text{million light years}} = \frac{22 \times 10^3\ m/s}{10^6 \times (9.4 \times 10^{12}\ km)}$$

($10^{12}\ km = 10^{15}\ m$)

$$= \frac{22 \times 10^3\ m/s}{9.4 \times 10^{21}\ m} = 2.3 \times 10^{-18}\ s^{-1}$$

$$\Rightarrow \boxed{H = 2.3 \times 10^{-18}\ s^{-1}}$$

$$\Rightarrow \rho_{critical} = \frac{3}{8(3.1415)} \frac{(2.3 \times 10^{-18}\ s^{-1})^2}{6.6743 \times 10^{-11} \frac{m^3}{kg \cdot s^2}}$$

($\frac{1}{s^2}$)

$$= 9.46 \times 10^{-27}\ kg/m^3$$

$$\rho_{critical} = 9.46 \times 10^{-27} \, kg/m^3 \simeq 10^{-26} \, kg/m^3$$

⇒ corresponds to a small grain of dust (mass $= 10^{-14} - 10^{-13}$ kg) per cube 10 km on the side.

~~The~~ As best we can tell the (average) density of the universe is $\simeq 10^{-26} \, kg/m^3$
(includes dark matter)