Wednesday, March 25, 2026

<u>Example</u>: $\boxed{\text{Rotation frequency}}$ of a sun-based white dwarf ...... and "neutron star".

$\boxed{\text{major assumption: no mass loss}}$

the Sun <u>cannot</u> produce a neutron star, but this is a "what if" exercise.

<u>note</u>: $T_{sun} = 25 \text{ days} = 2.16 \times 10^6 \text{ s}$

$$\hookrightarrow f_{sun} = \frac{1}{T_{sun}} = 4.63 \times 10^{-7} \text{ Hz}$$

According to the Radius vs. Mass plot for white dwarfs, a 1 solar mass white dwarf has a radius of

$$R_{wd} = 0.0078 \, R_{sun} = 5400 \text{ km}$$

For the neutron star scenario, we will use $R_{NS} = 14 \text{ km}$

$$= \frac{14}{6.96 \times 10^5} R_{sun}$$

$$\simeq 2 \times 10^{-5} R_{sun}$$

<u>IMPORTANT</u>: When the Sun contracts into a white dwarf (no mass loss), <u>angular momentum is conserved</u>.

Angular momentum $= L = mvr$ for a particle going in a circle

Also, $v = \frac{2\pi r}{\underset{\text{period}}{T}} = 2\pi r \underset{\text{frequency}}{f}$

$\Rightarrow L = 2\pi r^2 f m$

For a rotating sphere: $L \underset{\text{"proportional"}}{\propto} 2\pi r^2 f m$

or $L = \alpha\, 2\pi r^2 f m$

proportionality constant that accounts for the geometry of the sphere.

If angular momentum is conserved, then

$$\left.\begin{aligned} L_{sun} &= \alpha\, 2\pi\, r_{sun}^2 f_{sun}\, m \\ L_{wd} &= \alpha\, 2\pi\, r_{wd}^2 f_{wd}\, m \\ L_{ns} &= \alpha\, 2\pi\, r_{ns}^2 f_{ns}\, m \end{aligned}\right\}$$

with $L_{sun} = L_{wd} = L_{ns}$

(m remains unchanged by assumption)

Thus $\cancel{\frac{L_{sun}}{L_{wd}}}^{\,1} = \frac{\cancel{\alpha}\, \cancel{2\pi}\, r_{sun}^2 f_{sun} \cancel{m}}{\cancel{\alpha}\, \cancel{2\pi}\, r_{wd}^2 f_{wd} \cancel{m}} \Leftrightarrow \boxed{f_{wd} = \left(\frac{r_{sun}}{r_{wd}}\right)^2 f_{sun}}$

$\Rightarrow f_{wd} = \left(\frac{1}{0.0078}\right)^2 f_{sun} = (16437)(4.63 \times 10^{-7}) = 0.00761\ Hz$

$$\boxed{\begin{aligned} &\Rightarrow f_{wd} = 0.0076\ Hz \\ &\Leftrightarrow T_{wd} = \frac{1}{f_{wd}} = 131s = 2 \text{ minutes } 11 \text{ seconds} \end{aligned}}$$

neutron star: $\boxed{f_{ns} = \left(\frac{r_{sun}}{r_{ns}}\right)^2 f_{sun}} = \left(\frac{1}{2 \times 10^{-5}}\right)^2 f_{sun}$

$= \left(2.5 \times 10^9\right)\left(4.63 \times 10^{-7}\right)$

$= 1153\ Hz$

$$\Rightarrow \boxed{f_{ns} = 1153\ Hz \Leftrightarrow T_{ns} = \frac{1}{f_{ns}} = 0.86\ ms = 0.86 \times 10^{-3}\ s}$$

neutron star rotation rate

neutron star rotation period