

# Today's Topics

Friday, March 27, 2026 (Week 8, lecture 23) – Chapter 24.

**Problem Set #7** is due on ExpertTA on Monday, March 30, 2026, by 9:00 AM

**Problem Set #8** is due on ExpertTA on Friday, April 3, 2026, by 9:00 AM

**Midterm #2** is on Monday, April 6, 2026.

# Today's Topics

Friday, March 27, 2026 (Week 8, lecture 23) – Chapter 24.

A. Einstein's Theory of Relativity.

B. Special Relativity.

C. Length contraction.

D. Time dilation.

E. General Relativity.

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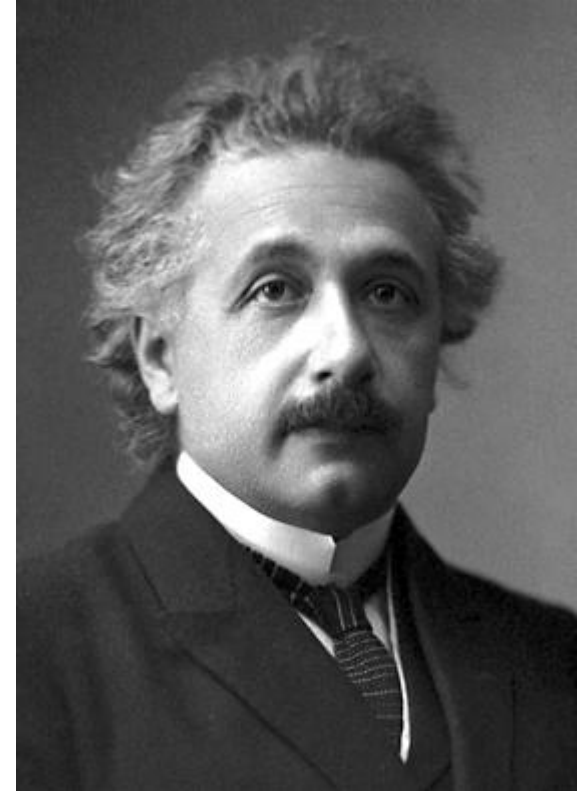
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# Einstein's Theory of Relativity

## 1905: Annus Mirabilis

- Brownian motion (motion of atoms in a gas).
- Photo-electric effect (discovery of the photon,  $E = hf$ )
- **Special theory of relativity.**
  - Major revision of Galilean relativity.
  - Equivalence of energy and matter:  $E = mc^2$



Albert Einstein, 1921.  
(1879-1955)

# Einstein's Theory of Relativity

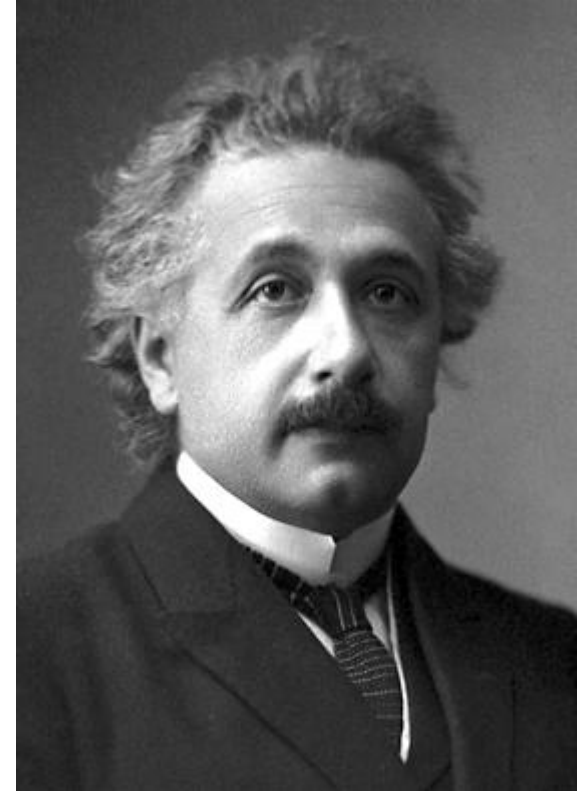
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## 1907-15: General Relativity

Theory of relativity applied to gravity.

- gravity = curved space-time.



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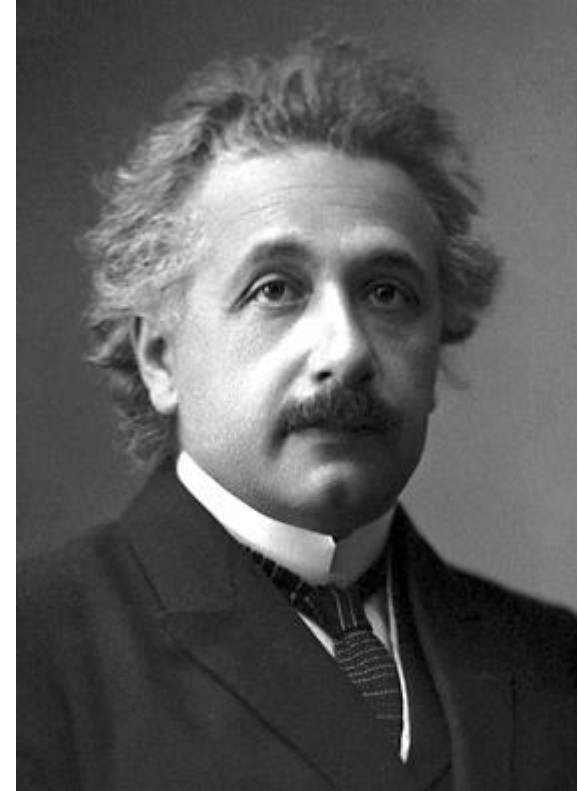
→ gravity = curved space-time.

1921: Nobel Prize for photo-electric effect.

1924: Bose-Einstein Condensation

Predicts the existence of a new type of quantum matter.

- Builds on the work of Satyendra Bose.
- First observed in 1995
- There is a BEC in the basement of Small Hall (room # 069).



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# Inertial Frames (Galileo & Einstein)

## Inertial Frame

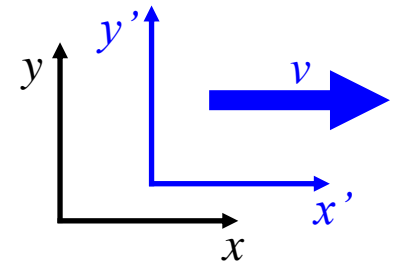
Coordinate system at constant velocity in a rest frame.

*think of it as a box*

## Rest Frame

A coordinate system that is not moving.

*Note: a rest frame is an inertial frame.*



# Inertial Frames (Galileo & Einstein)

## Inertial Frame

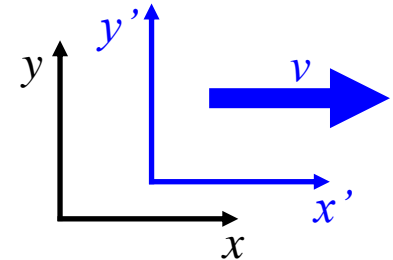
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## Important

- **You cannot tell if you are moving** based on local measurements inside your inertial reference frame (the frame attached to you).
- If you are **accelerating/decelerating**, then you can tell based on local measurements (i.e. there is a force on you that you can measure,  $F = ma$ ).

# Special Relativity (Einstein)

## Principle of Relativity

The laws of physics are the same in all inertial reference frames.

## Corollary #1

You cannot tell if you are moving (based on local measurements) in an inertial frame.

## Corollary #2: Universal speed of light

The speed of light in vacuum is the same in all inertial frames, regardless of the motion of the source.

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Length contraction & time dilation

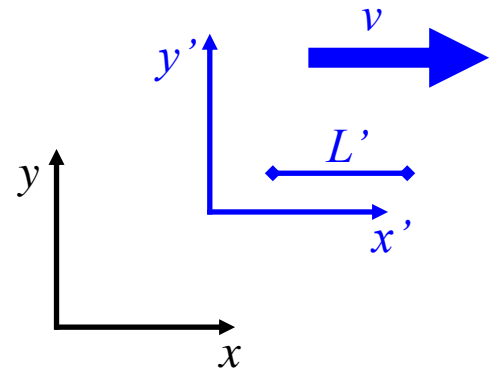
# Special Relativity

## Length Contraction

### In the $x'$ - $y'$ inertial frame

Consider a rod of length  $L' = L_0$ , as measured in the  $x'$ - $y'$  inertial frame (i.e. the rest frame of the rod).

*Note: The rod is aligned with the axis of motion along  $x'$ .*



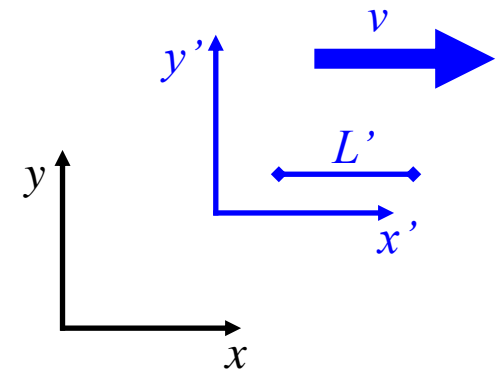
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### In the $x$ - $y$ inertial frame

If you measure the length of the rod, then you will

get a shorter length:  $L = \frac{L_0}{\gamma}$ .

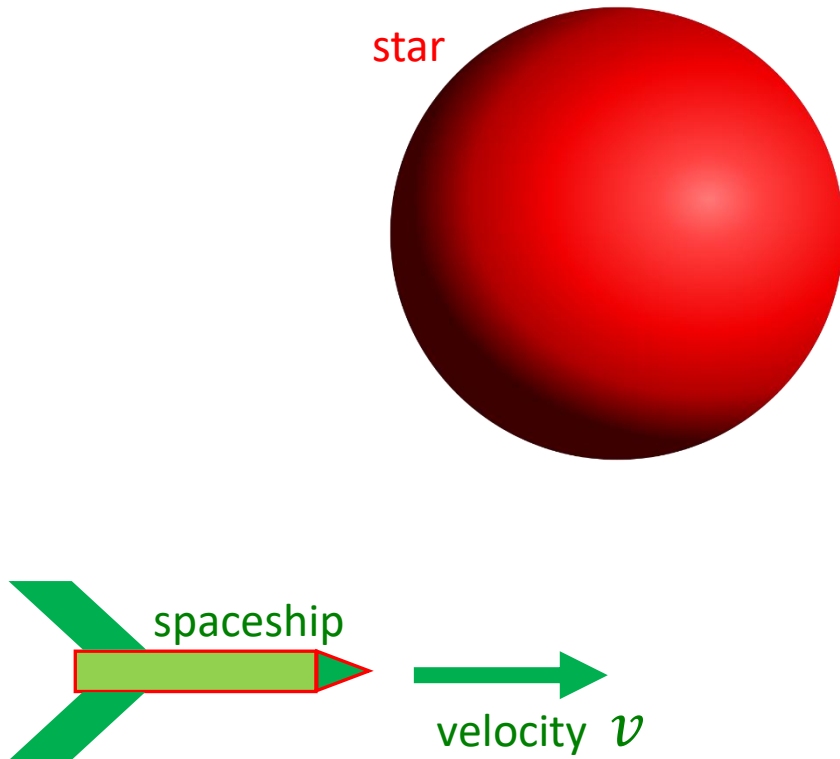
Gamma factor:  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\gamma \geq 1$$

**Note:** the length contraction is only along the axis of motion. Along axes perpendicular to the motion, there is no change in length.

# Length Contraction: Example

Consider a spaceship travelling past a spherical star at 90% of the speed of light.

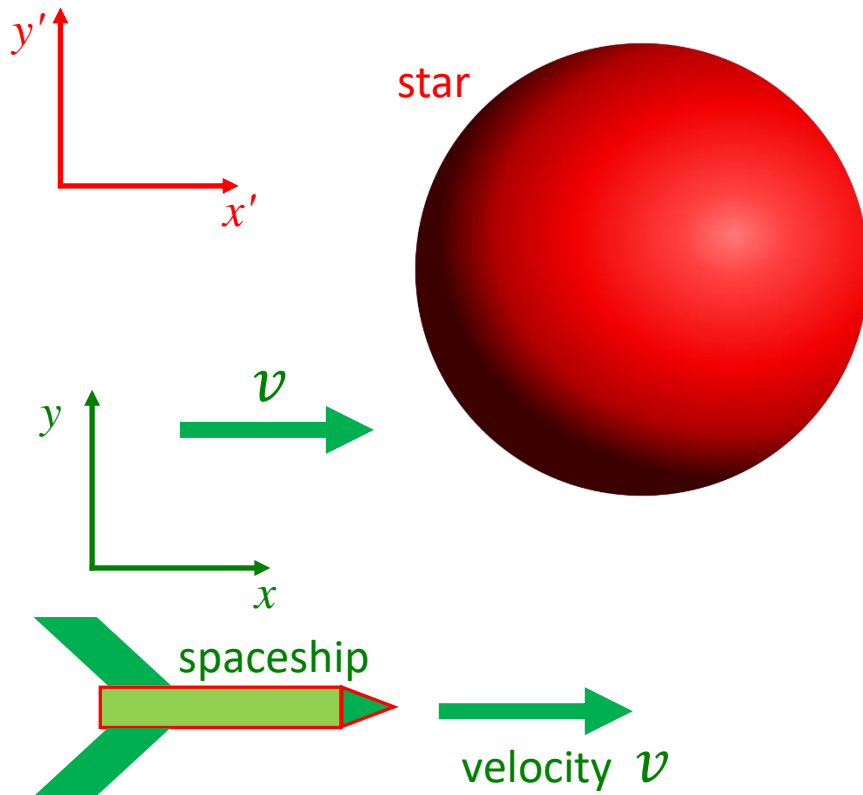


**Question:** What is the shape of the star in the frame of the spaceship?

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Rest frame of the star

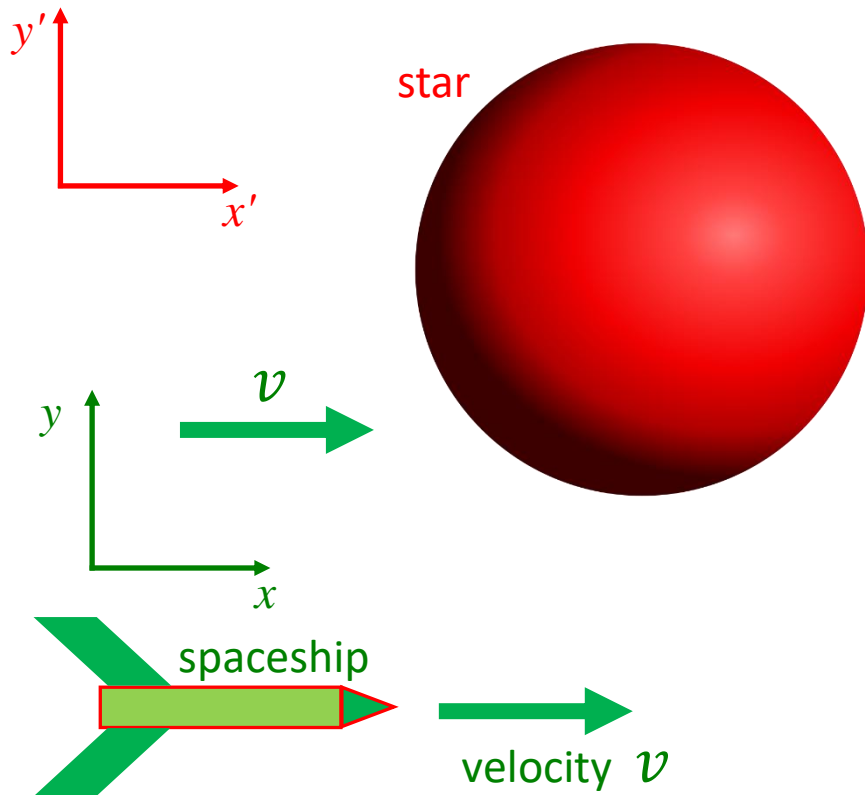


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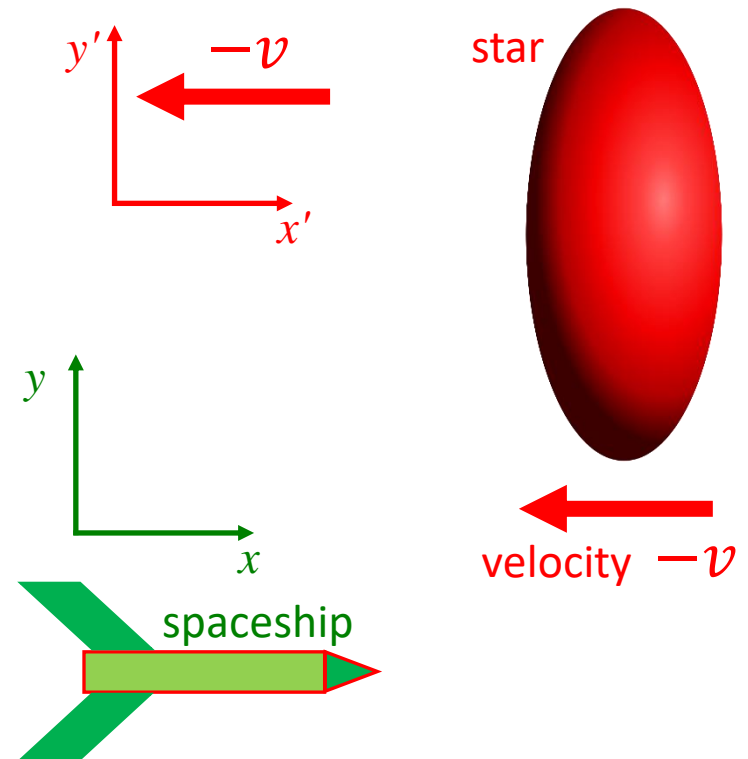
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## Rest frame of the star



**Question:** What is the shape of the star in the frame of the spaceship?

## Rest frame of the spaceship

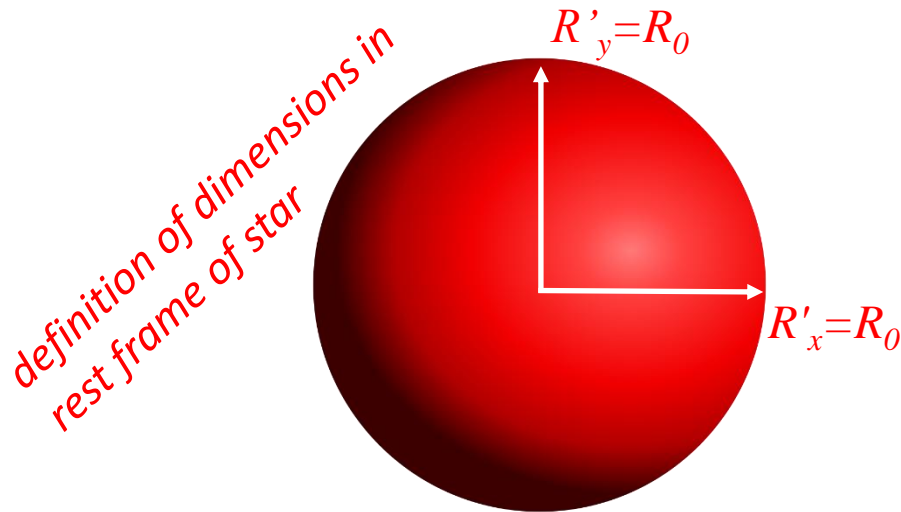


**Answer:** The star appears/is compressed along the axis of travel.  
*The transverse directions are unaffected.*

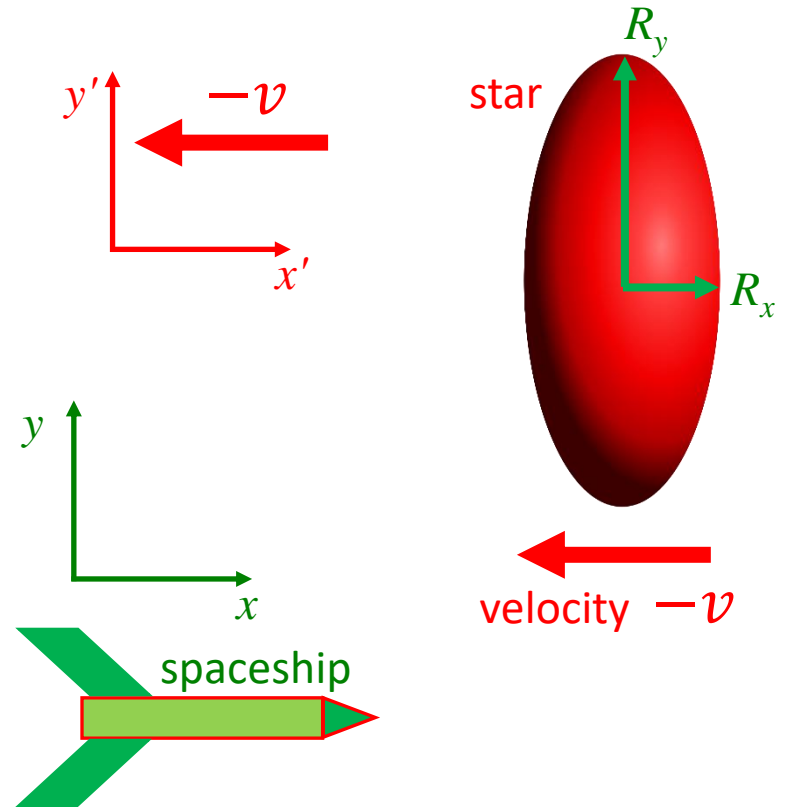
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## Quantitative answer



## Rest frame of the spaceship



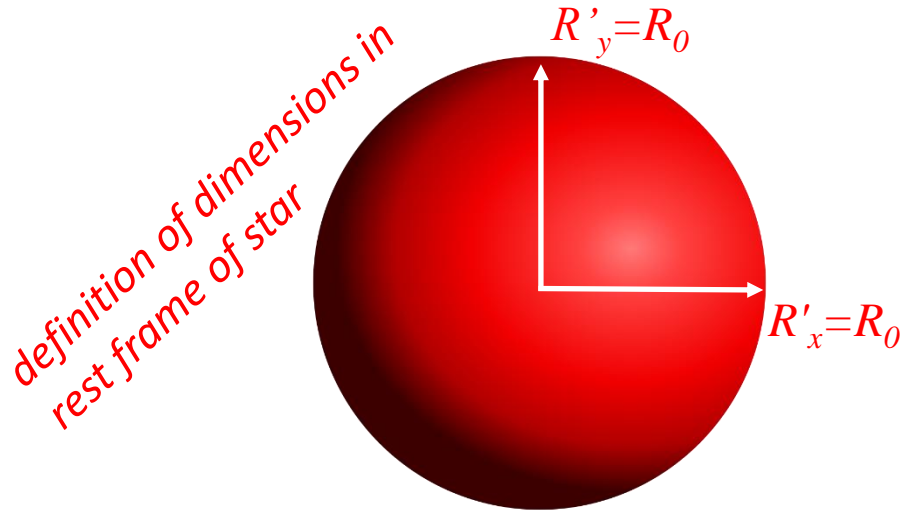
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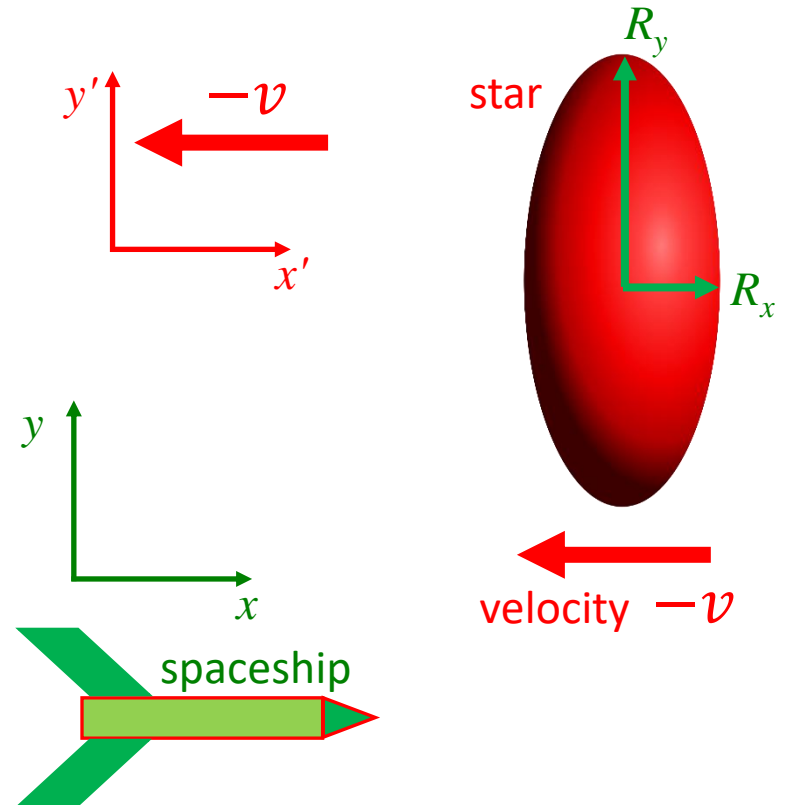
## Quantitative answer



In the rest frame of the spaceship, we have

$$R_x = \frac{R_0}{\gamma} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Rest frame of the spaceship



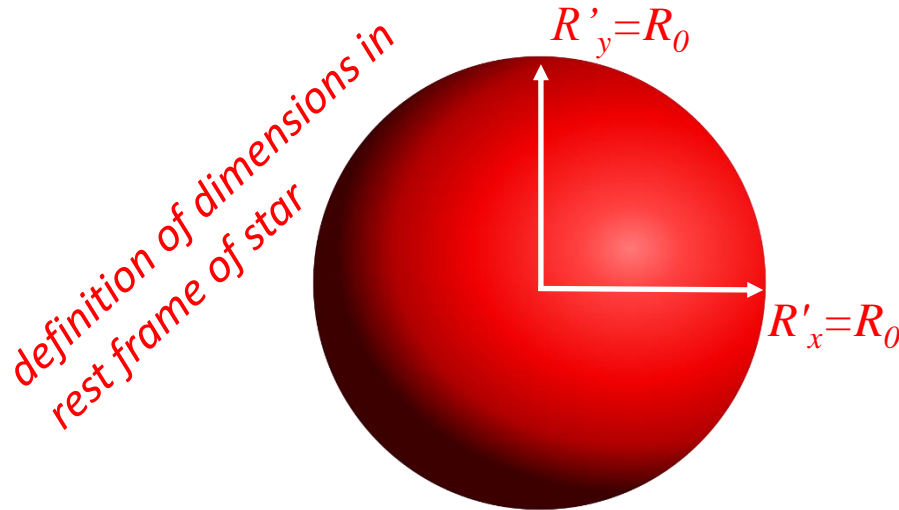
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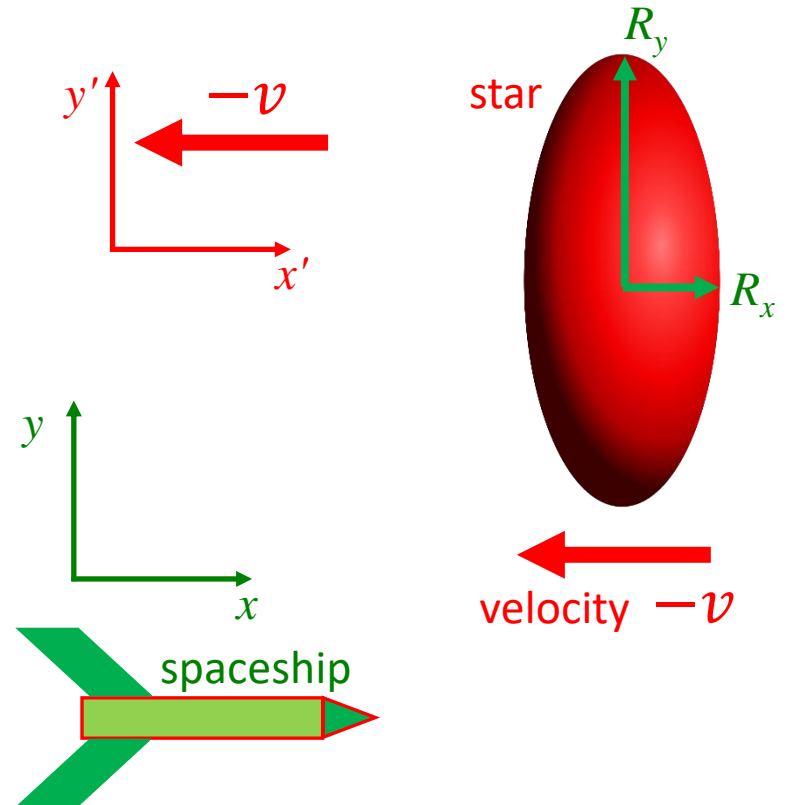
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In the rest frame of the spaceship, we have

$$R_x = \frac{R_0}{\gamma} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(-0.9c)^2}{c^2}}}$$
$$= \frac{1}{\sqrt{1 - 0.81}} = 2.29$$

## Rest frame of the spaceship



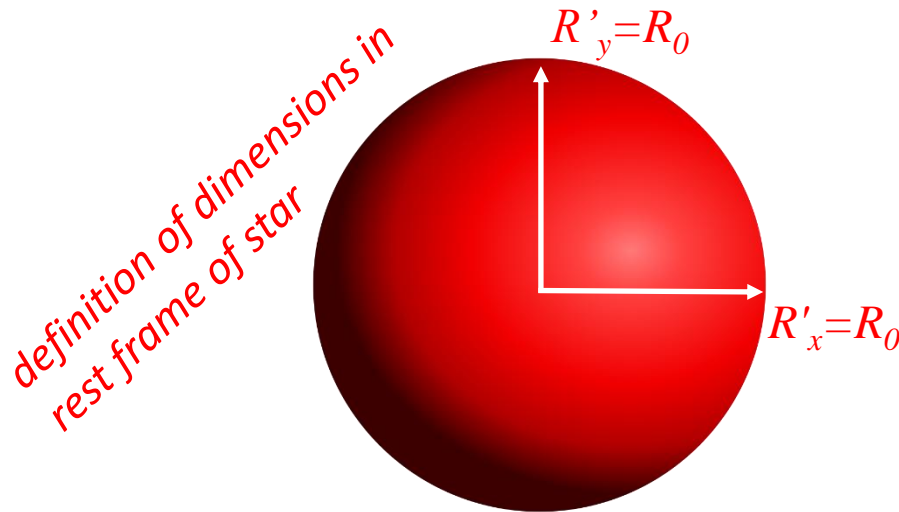
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Consider a spaceship travelling past a spherical star at 90% of the speed of light.

## Quantitative answer

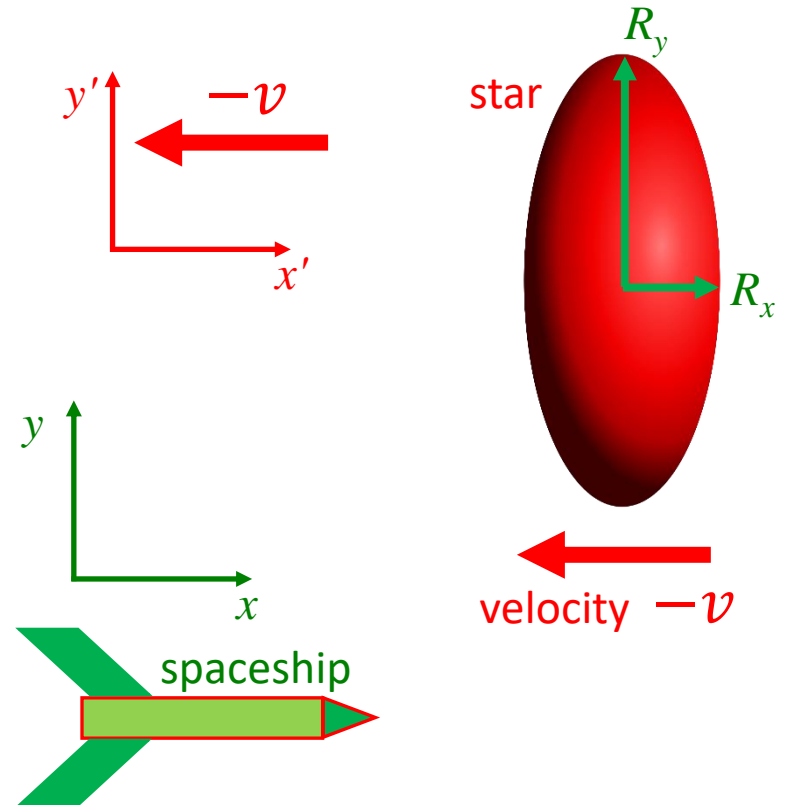


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$$\text{Thus } R_x = \frac{R_0}{2.29} = 0.43R_0$$

## Rest frame of the spaceship



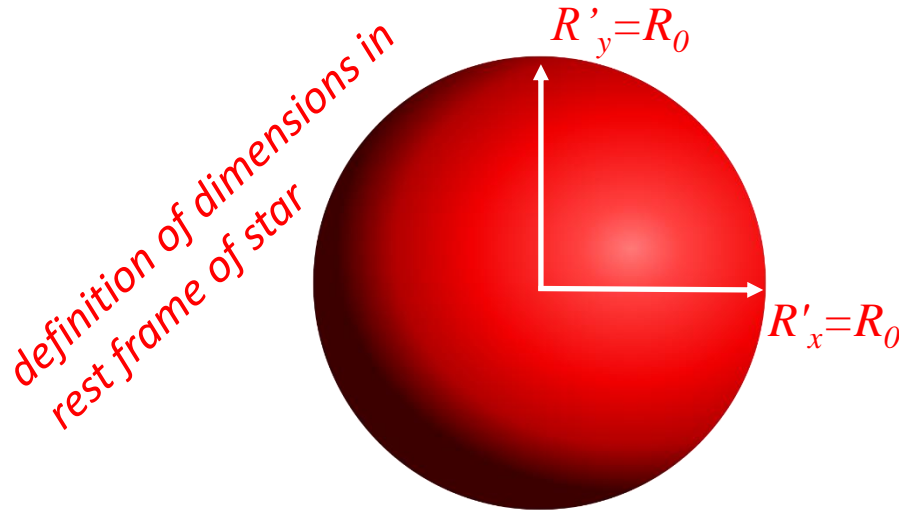
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*The transverse directions are unaffected.*

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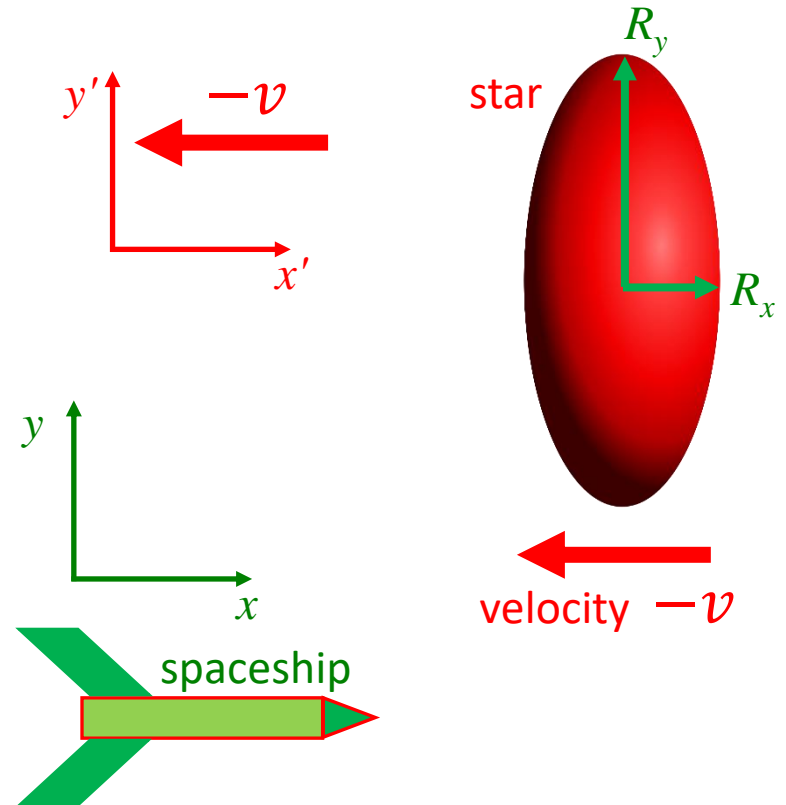
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$$= \frac{1}{\sqrt{1 - 0.81}} = 2.29$$

Thus  $R_x = \frac{R_0}{2.29} = 0.43R_0$

## Rest frame of the spaceship



**Answer:** The star appears/is compressed to 43% of its original size along the direction of travel.  
*The transverse directions are unaffected.*

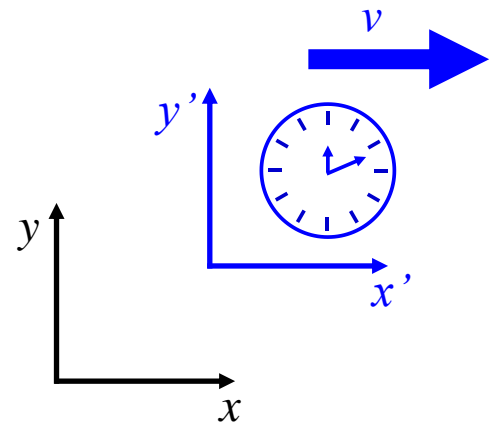
**PolEv Quiz: [PolEv.com/sethaubin](https://PolEv.com/sethaubin)**

# *Special Relativity*

## **Time Dilation**

### **In the $x'$ - $y'$ inertial frame**

Consider a clock at rest in the  $x'$ - $y'$  inertial frame that measures a time interval of  $\Delta T' = T_0$ , i.e. the time for the big clock hand to go from noon to the 2 o'clock position (10 minutes).



# Special Relativity

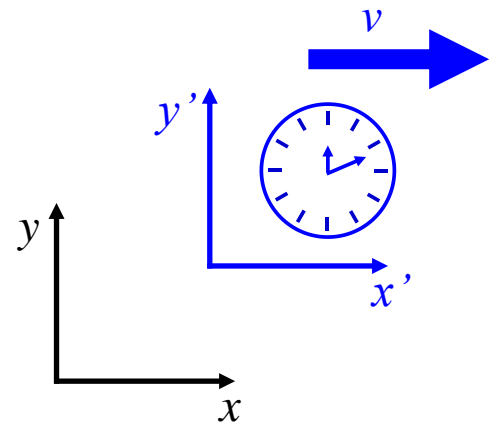
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### In the $x$ - $y$ inertial frame

If you measure the same elapsed time (with your own timepiece) from the  $x$ - $y$  inertial frame, i.e. as the clock flies past you at speed  $v$ , then you will measure a longer elapsed time:  $T = \gamma T_0$ .



# Time Dilation Example

## The Twin Paradox

- Twin A travels to a distant star at a velocity of  $v = 0.9c$  and then returns also at a velocity  $v = 0.9c$ , while twin B remains on Earth.
- Twin A measures a travel time of 10 years (according to twin A's clock) to get to the star, and then 10 years to return to Earth.

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How much older is twin A, when twin A returns to Earth?

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### Question 1

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### Answer 1

Since we are using twin A's clock, we know that

$$\Delta T' = T_0 = 2 \times 10 \text{ years} = 20 \text{ years}$$

Twin A has aged 20 years (in the physics-biology sense).

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### Question 2

How much older is twin B, when twin A returns to Earth?

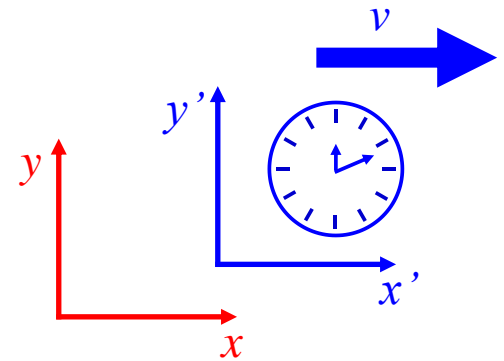
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### Question 2

How much older is **twin B**, when **twin A** returns to Earth?



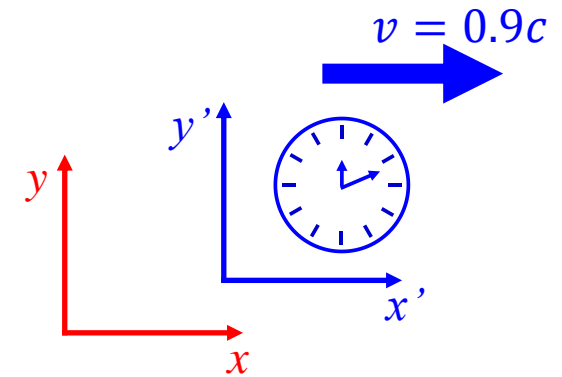
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### Answer 2

If **twin B** is in the **x-y frame (Earth)**, and **twin A** is in the **x'-y' frame (spaceship)**, then

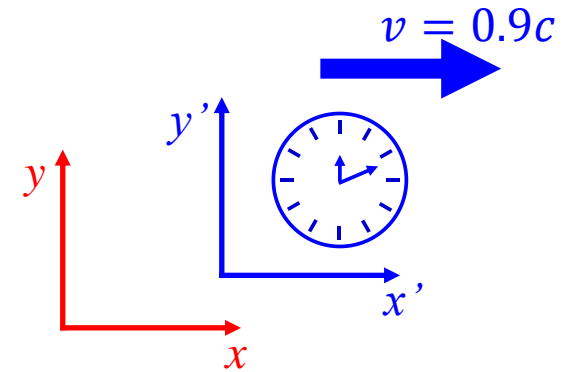
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$$\Delta T = \gamma \Delta T' = \gamma T_0 = 2.29 \times 20 \text{ years} = 45.8 \text{ years}$$

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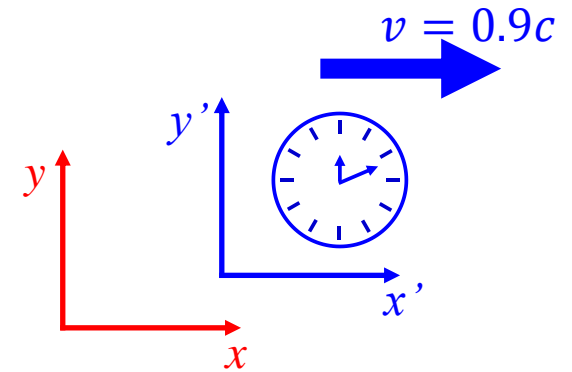
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Twin B has aged 45.8 years while remaining on Earth !!!

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Twin A sees twin B travelling away from the spaceship on “spaceship Earth”, so why doesn't twin A age faster instead?

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### Answer 3

Twin A must accelerate and decelerate, so twin A is briefly in a **non-inertial frame**. The motions of twin A & twin B are not symmetric.

# General Relativity

## Equivalence Principle

A coordinate system that is falling freely in a gravitational field is (equivalent to) an inertial frame.

## Corollary

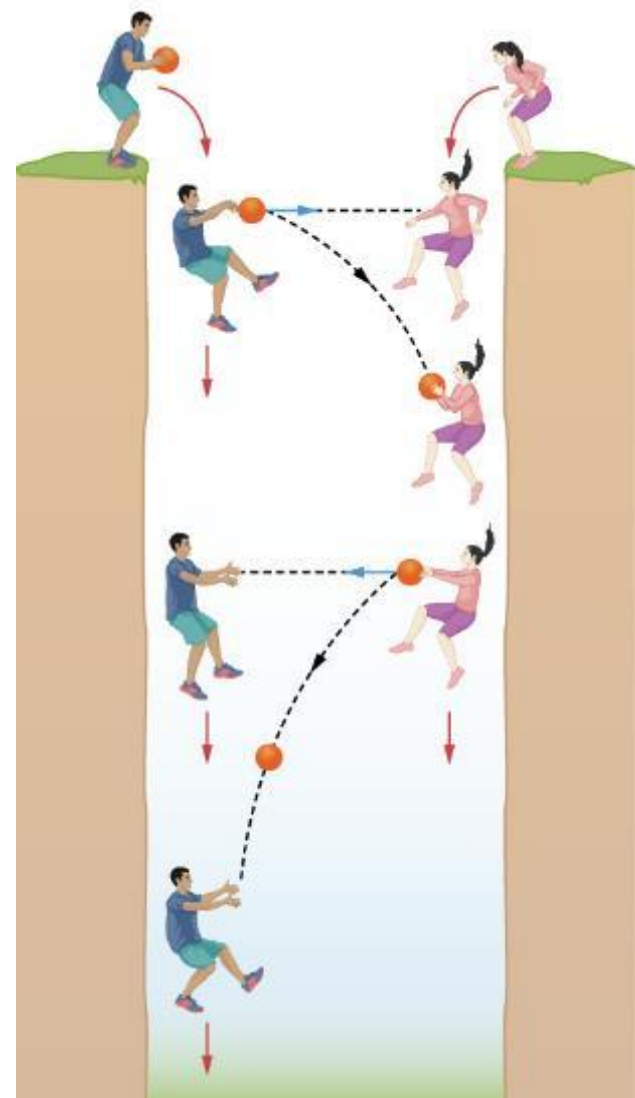
You cannot tell if you are at rest in a non-gravitational field (i.e. in a standard inertial frame) or freely falling under gravity based on local measurements.

# Equivalence Principle

You cannot tell if you are at rest in free space (i.e. in a standard inertial frame) or freely falling under gravity based on local measurements.

## Example

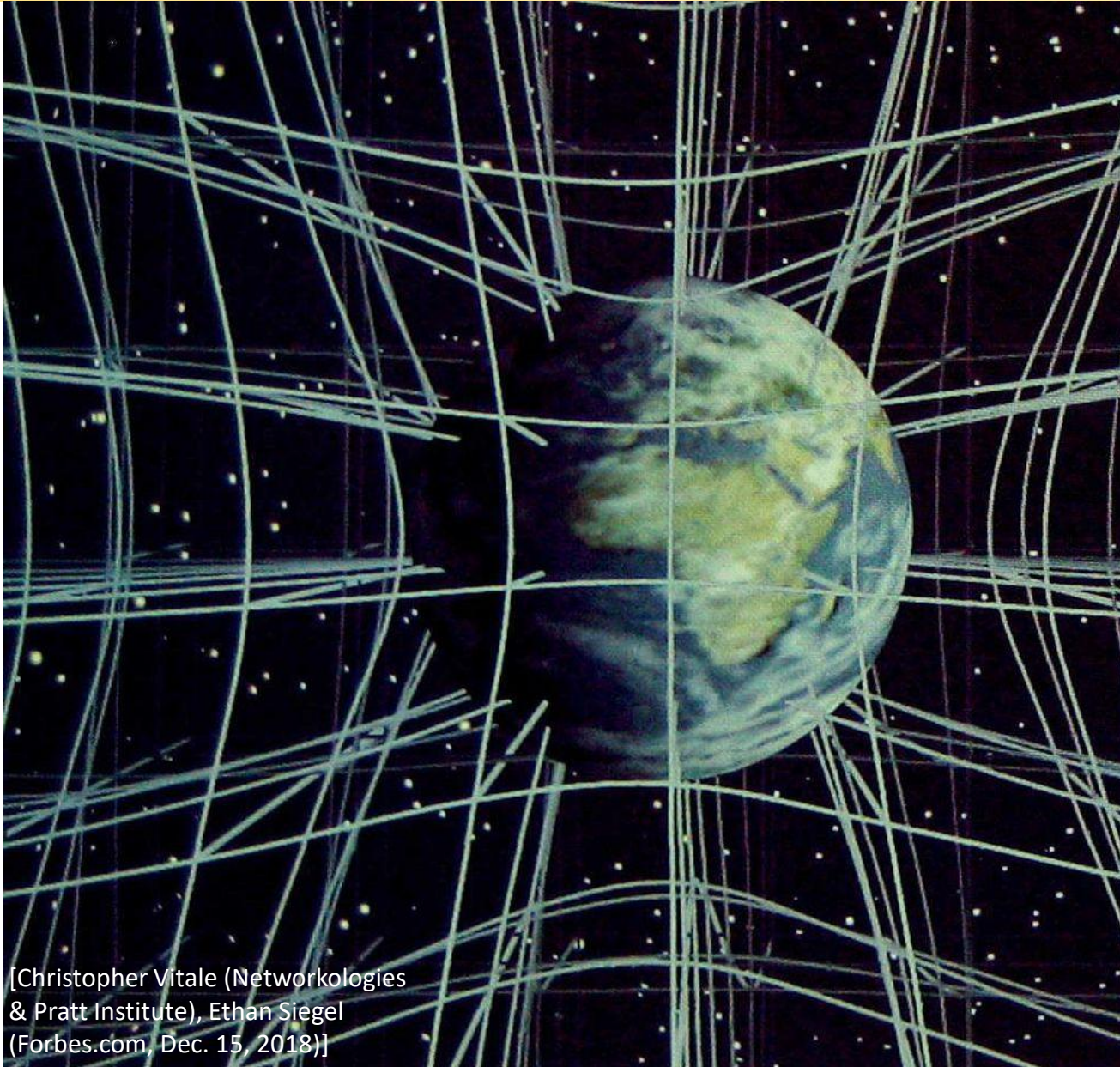
- Two people play catch as they descend into a bottomless abyss.
- Since the people and ball all fall at the same speed, it appears to them that they can play catch by throwing the ball in a straight line between them.
- Within their frame of reference, there appears to be no gravity.



# Equivalence Principle on ISS



# Curved Space-Time



[Christopher Vitale (Networkologies  
& Pratt Institute), Ethan Siegel  
(Forbes.com, Dec. 15, 2018)]

# Curved Space-Time: light rays in 2D

The gravity of a massive object bends the fabric of space and time.

