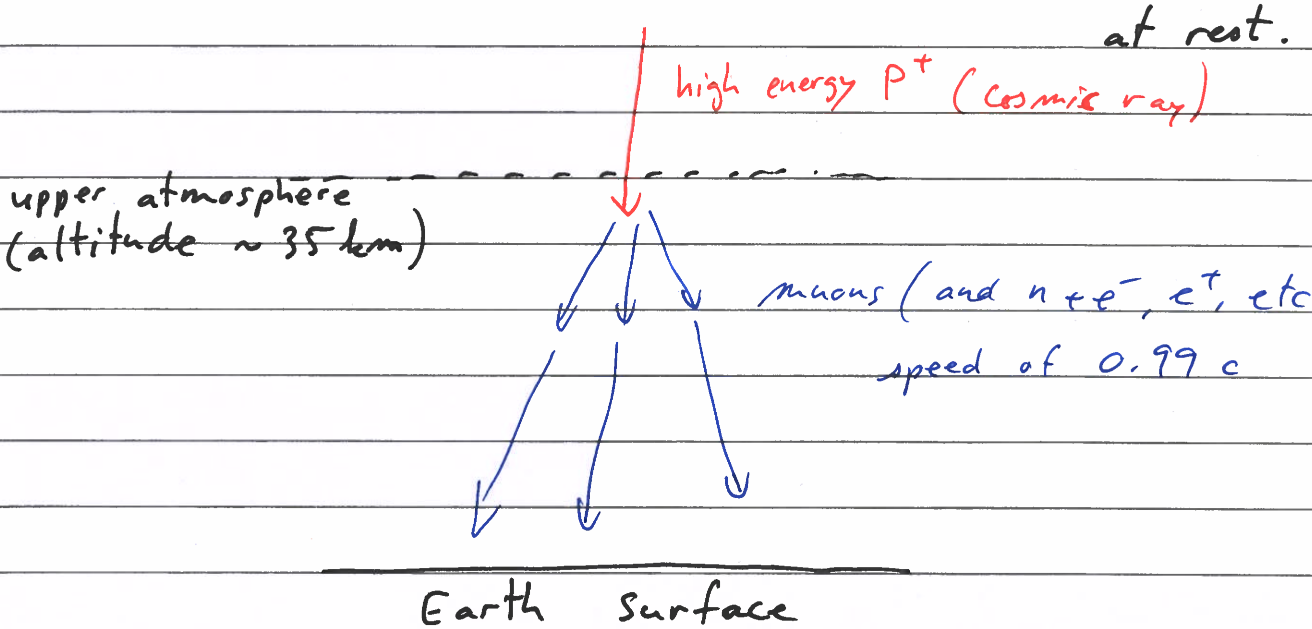
Monday, March 30, 2026

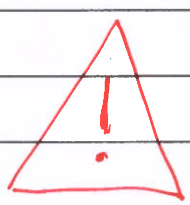
Example : Muons from Cosmic Rays

A muon (μ^-) is a "heavy electron" with the same charge as an electron (e^-), but with 207 times the mass of an electron.

Muons are unstable and have a lifetime of 2.2 μs at rest.



Galilean "naive" travel distance



Wrong

$$\Delta z = \Delta t \times v = (2.2 \times 10^{-6}\, s)(\underbrace{0.99 \times 3 \times 10^{8}\, m/s}_{2.97 \times 10^{8}\, m/s})$$

$$= 653\ m$$

Experimental fact: Cosmic ray muons are observed in large numbers on Earth's surface.

Special Relativity Calculation of travel distance

Muons "live" for 2.2 $\mu s = \Delta t'$ in their rest frame

In our frame (Earth's frame), the muon lifetime experiences time dilation:

$\Delta t = \gamma \, \Delta t'$ with $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ with $v = 0.99c$

(Δt: in Earth's frame; $\Delta t'$: in muon's rest frame)

$$= \frac{1}{\sqrt{1-\left(\frac{0.99\,c}{c}\right)^2}}$$

$$= \frac{1}{\sqrt{1-0.9801}}$$

$$= 50.3$$

In Earth's frame, the muon's lifetime (and travel time) is

$\Delta t = 50.3 \times 2.2\ \mu s = 110.66\ \mu s \simeq 110 \times 10^{-6}\ s$

$\Rightarrow$ $\boxed{\Delta t \simeq 110 \times 10^{-6}\ s}$

Vertical travel distance of muons: $\Delta z = v \times \Delta t$

$= (2.97 \times 10^{8}\ m/s)(110 \times 10^{-6}\ s)$

$= 32\,834\ m$

$\simeq 33\ km$

$\Rightarrow$ $\boxed{\Delta z \simeq 33\ km}$

$\Rightarrow$ Many muons should make it to the Earth's surface.