Friday, April 3, 2026

## Example: Time dilation near a black hole

Q: How much slower does a clock run when it is 1 m from the event horizon of a black hole?

i.e. How much faster does the rest of the universe tick?

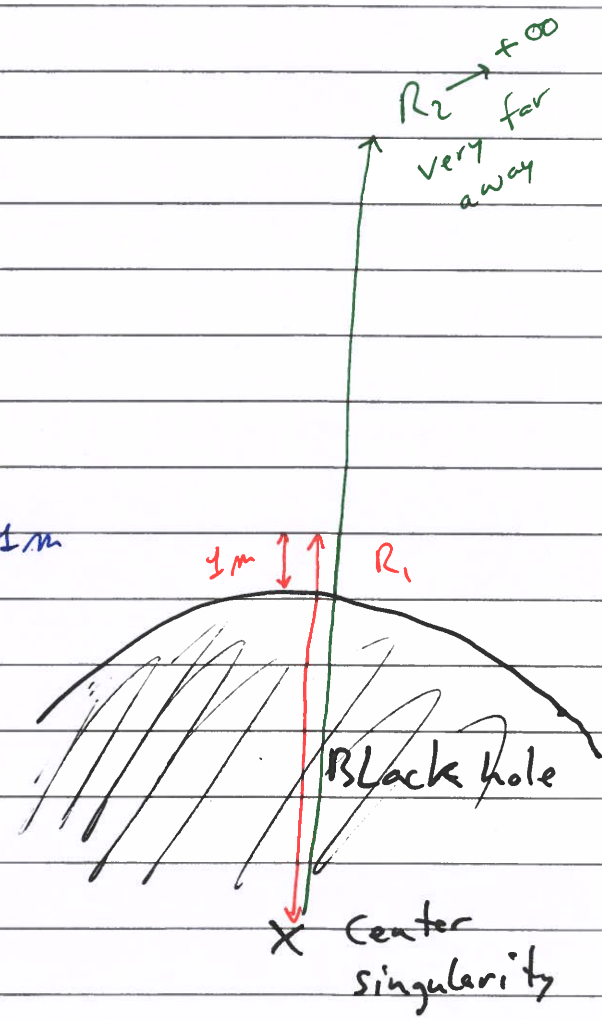
Black hole at the center of the Milky Way (Sagittarius A*)

$$R_s = 17\ R_{sun} = 12 \times 10^6\ km = 12 \times 10^9\ m$$

$$\frac{\text{clock 2 frequency}}{\text{clock 1 frequency}} = \frac{f_2}{f_1}$$

$$= \sqrt{\frac{1 - \cancel{R_s/R_2}^{\,0}}{1 - R_s/R_1}} \leftarrow R_1 = R_s + 1m$$

$$= \sqrt{\frac{1}{1 - R_s/R_1}}$$



$$= \sqrt{\frac{1}{\frac{R_1}{R_1} - \frac{R_s}{R_1}}} = \sqrt{\frac{1}{\frac{R_1 - R_s}{R_1}}}$$

$$= \sqrt{\frac{R_1}{\underbrace{R_1 - R_s}_{= 1\,m}}} = \sqrt{\frac{12 \times 10^9 + 1}{1}}$$

$$\simeq \sqrt{12 \times 10^9}$$

$$= 1.095 \times 10^5$$

$$\simeq 110{,}000$$

$$\boxed{\text{At 1 m from the event horizon, the rest of the universe evolves at a rate 110,000 times faster than you.}}$$

note: At 1 m from the event horizon of the M87 black hole, the rest of the univers evolves at a rate $4.4 \times 10^6$ times faster than you.

$$[\, R_{s_{M87}} = 128 \text{ AU} = 1.92 \times 10^{13} \text{ m} \,]$$