A couple cool facts about Statistical Mechanics

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Abstract

This document covers a few examples of the power of LaTeXand will hopefully be a touch informative.

1 Introduction

Ludwig Boltzman, who spent much of his life studying statistical mechanics, died in 1906, by his own hand. Paul Ehrenfest, carrying on the work, died similarly in 1933. Now it is our turn to study statistical mechanics. Perhaps it will be wise to approach the subject cautiously. (Opening lines of "States of Matter", by D.L. Goodstein).

2 Images, data, and charts, oh my!

2.1 Images

Entropy says that the universe tends towards chaos - i.e., energy is always lost as heat, which cannot be controlled or directed without expending a larger amount of energy.

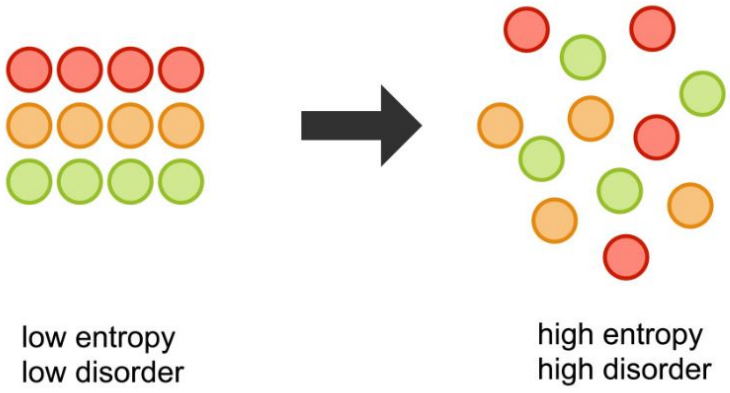

Figure 1: Chaos visualized [Cha].

2.2 Cool tables

In our system, we have neutrons and protons freely moving about a box. They form what is known as an **ensemble** - a collection of particles, each acting with individual motion. Sometimes, the neutrons and protons will collide, producing Deuterium. We begin with 10 neutrons and 10 protons. We observe that, for time measured in seconds, we have

Time(in s)	Deuterium (in atoms)
1	0
2	4
3	7
4	8
5	9

Table 1: An example table.

3 Methodology

3.1 Mathematical Methods

We can describe the energies of the states with the following equation

$$E = \sum_k e^{\frac{E_k}{(k_b T)}}$$

We can now use the discrete energies given by the previous equation to find the energy of the system.

Let $X_1, X_2, \ldots, X_n$ be individual particles in our ensemble, with $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}[X_i] = \sigma^2 < \infty$, and let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{1}{n}\sum_i^n X_i$$

denote their mean. We can easily find the total energy by multiplying by that mean. However, in most systems, you don't know exactly how many atoms you will have, so you want to allow for what is known as **variance**, which is *the range of uncertainty of a given ensemble*

Then as n approaches infinity, the random variables $\sqrt{n}(S_n - \mu)$ converge in distribution to a normal $\mathcal{N}(0, \sigma^2)$.

3.2 How to add Lists

We have now "covered" ...

1. The nature of entropy
2. and a basic ensemble

... as well as ...

- Mathematical methods
- and not much else

3.3 Good luck!

I hope you've found this lesson educational, if not somewhat misleading, considering I haven't studied stat mech in over two years. Anyway if you want to know more about overleaf, click this link. Please also let us know if you have any questions or feedback about this. I hope it was both educational and enjoyable.

References

[Cha] Shreya Chakraborty. What is the relationship between thermodynamics and statistical mechanics?