Measuring Graphite Layer Spacing with Electron Diffraction

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Abstract

In this experiment, we used an electron gun to measure the diffraction from graphite and calculate calculate the inter-atomic layer spacing. The best spacing from the inner diffraction ring was $d_{10} = 0.135 \pm 0.007$ nm and the best spacing from the outer ring was $d_{11} = 0.235 \pm 0.012$ nm. We rejected both calculations as the literature values are outside the uncertainty. We attribute the uncertainty to measurement error in the procedure to measure arc length and to errors in the voltage readings.

1 Theory overview

The main physical phenomenon at play is diffraction from the wave nature of electrons. An electron wave passing through the first layer in the graphite and reflecting off the second has the same angle of reflection as one reflecting off the first layer, as shown in Figure 1.1 of the lab manual. The two waves then interfere, causing a concentric ring pattern on the screen. The de Broglie wavelength of the electron, $\lambda = h/p$ where momentum p is, according to [1],

$$\frac{1}{2}m_e v^2 = \frac{p^2}{2m_e} = eV_a \tag{1}$$

where V_a is the voltage accelerating the electron. This can be combined with Bragg's Law governing constructive interference

$$n\lambda = 2d\sin\theta \tag{2}$$

where $n = 0, 1, 2...$ for each order diffraction. θ is the angle of deflection, calculated by

$$\theta(R, L, s) = \frac{1}{2}\arcsin\left(\frac{R}{L}\sin\left(\frac{s}{2R}\right)\right) \tag{3}$$

where R is the radius of curvature of the screen, L is the distance between the screen and target, and s is the arc length between edges of the circles. Thus, the dependence of interest from reference [1] for the inter-atomic spacing seen with electron diffraction is given by

$$d(\theta, V_a) = \frac{h}{2\sin\theta\sqrt{2em_eV_a}} \tag{4}$$

where h is Planck's constant, e is the electron charge, m_e is the electron mass, and V_a and θ are defined above. The n becomes 1 and is left out because we are only focusing on the spacing between the first layers, so we only need the first order diffraction.

2 Experimental setup and procedures

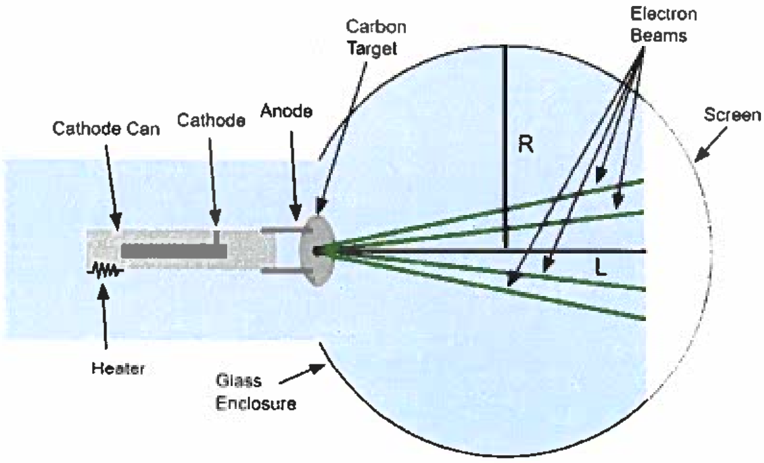

Figure 1: Internal diagram for Electron Diffraction apparatus. R = .064 m is the radius of curvature of the screen. L = .130 m is the distance to the screen from the target.

We set up the electron diffraction apparatus, connected it to a power supply and turned it on to warm up. First, we used a ruler to measure R and L on the apparatus. We adjusted the voltage to find a lower and upper bound of V_a for which the diffraction pattern is clearly visible on the screen. Then, we wrapped receipt paper around the apparatus and used a pencil to mark the edges of the inner and outer circles. We used a ruler to measure the arc lengths marked off on the paper, with the outer markings corresponding to the outer ring and the inner ones for the inner ring. Then, we repeated the procedure at eight voltages V_a within the range.

3 Experimental data and data analysis

Table 1: Average inter-atomic spacing between graphite layers

Atomic Plane (Ring)	$< d >$ (nm)	σ (nm)
d_{10} (Inner)	0.135	0.007
d_{11} (Outer)	0.235	0.012

The average spacings and standard deviations in Table 1 are calculated by using the individual arc lengths to calculate the deflection with equation 3 and then using those angles in equation 4 to find the spacings. Hence, the mean spacings with their respective statistical uncertainties from Table 1 are $d_{10} = 0.135 \pm 0.007$ nm and $d_{11} = 0.235 \pm 0.012$ nm.

3.1 Error propagation

For determining error propagation, we used an error of δV_a =150 V for the power supply, mostly from fluctuations in and limitations on the built-in multimeter display. The angle error for both circles was calculated with

$$\delta\theta = \frac{s\cos\left(\frac{s}{2R}\right)}{4\sqrt{L^2 - R^2\sin^2\frac{s}{2R}}} * \frac{\delta s}{s} \tag{5}$$

as derived in Appendix A. The uncertainty in the arc length measurements, $\delta s = \pm 0.005m$ comes from ruler limitations and issues projecting the spherical distance to a measurable flat line. Using this, $\delta\theta_{inner} = 0.01$ radians and $\delta\theta_{outer} = 0.006$ radians. Thus, the error propagation for the inter-atomic spacing is calculated by

$$\delta d(V_a, \theta) = d_e\sqrt{\frac{1}{4}\left(\frac{\delta V_a}{V_a}\right)^2 + \left(\frac{\delta\theta}{\theta}\right)^2} \tag{6}$$

as derived in Appendix B. Therefore, the equation gives uncertainties of $\delta d_{11} = \pm 0.001$ nm and $\delta d_{10} = \pm 0.002$ nm, lessthan the standard deviation statistical error in both cases.

3.2 Graphical analysis

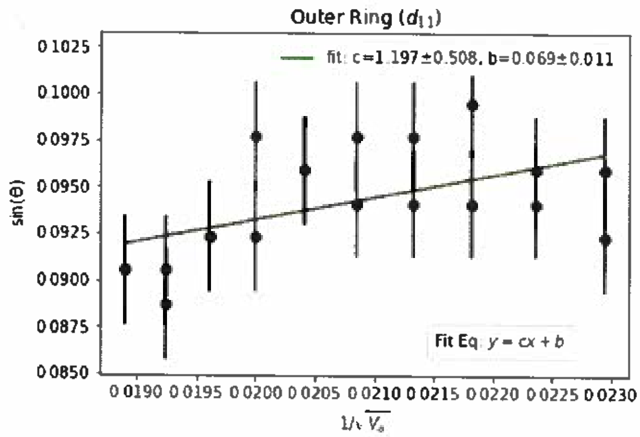

Figure 2: The graph shows $\sin(\theta)$ plotted against $1/\sqrt{V_a}$ for the outer ring. The slope of the best fit is c and the vertical intercept is b. The vertical lines represent the standard deviation in $sin(\theta)$.

The line of best fit in the graph above has the equation $\sin\theta = c\frac{1}{\sqrt{V_a}} + b$ where

$$c = \frac{h}{2\sqrt{2m_e e}} * \frac{1}{d} \tag{7}$$

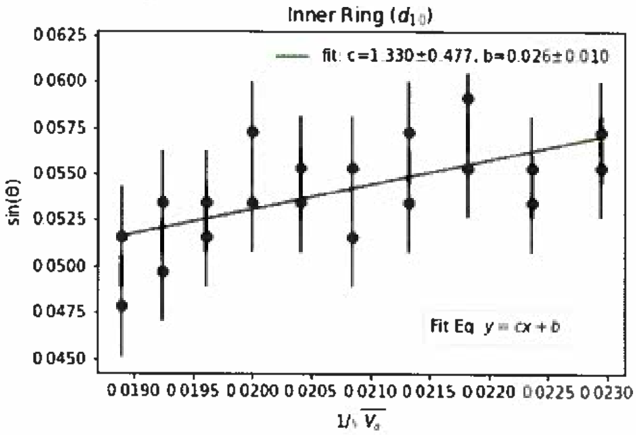

Figure 3: The graph shows $\sin(\theta)$ plotted against $1/\sqrt{V_a}$ for the inner ring. The best fit slope is c and vertical intercept is b. The vertical lines show the standard deviation in $sin(\theta)$.

which results in $d_{10} = 0.461$ nm and $d_{11} = 0.512$ nm. The uncertainty in these values is propagated through the equation as derived in Appendix C with the equation being

$$\delta d = d * \frac{\delta c}{c} \tag{8}$$

so that when we use the slope and uncertainty values from the graph, we get $\delta d_{10} = \pm 0.17$ nm and $\delta d_{11} = \pm 0.2$ nm. These values are much higher because the graphs are more affected than the averages by outliers. Both graphs show that lower voltages measurements were smaller than expected and high voltage ones were larger than expected, flattening the line.

The intercepts on the graphs should be zero theoretically but are above that, and zero is not within either uncertainty range. Both reflect systematic errors where the values for θ are larger than expected across the board. This means that out measurements of the arc length s for both rings were systematically larger than expected, an error likely stemming from the procedure of wrapping receipt paper around the apparatus and marking it with pencils.

4 Conclusions

The arithmetically estimated inter-atomic spacings were $d_{10} = 0.135 \pm 0.007$ nm and $d_{11} = 0.235 \pm 0.012$ nm and the graphically estimated spacings were $d_{10} = 0.46 \pm 0.17$ nm and $d_{11} = 0.5 \pm 0.2$ nm. We reject both sets of calculations as the literature values are outside the uncertainty range. The error in the measurement comes from flat projection error in the arc length which propagated through the angle calculation to the spacing calculation and the errors in the readings from the high voltage power supply.

References

[1] J. R. Stevens. *Physics 251 Atomic Physics Lab Manual*, 2016-2018 edition, 2018.

Appendices

A Derivation for Error Propagation in Equation 5

To find the error $\delta\theta$ in the angle we calculated from the orthodromic distance measured on the apparatus, we need to use the partial derivatives of equation 3 in the general error propagation formula

$$(\delta\theta)^2 = \left(\frac{\partial\theta}{\partial R}\right)^2(\delta R)^2 + \left(\frac{\partial\theta}{\partial L}\right)^2(\delta L)^2 + \left(\frac{\partial\theta}{\partial s}\right)^2(\delta s)^2 \tag{A.1}$$

The three partial derivatives for equation 3 are

$$\frac{\partial\theta}{\partial R} = \frac{1}{2L\sqrt{1 - \frac{R^2\sin^2\left(\frac{s}{2R}\right)}{L^2}}} * \left(\sin\left(\frac{s}{2R}\right) - \frac{s\cos\left(\frac{s}{2R}\right)}{2R}\right) \tag{A.2}$$

$$\frac{\partial\theta}{\partial L} = \frac{1}{2L\sqrt{1 - \frac{R^2\sin^2\left(\frac{s}{2R}\right)}{L^2}}} * \frac{-R\sin\left(\frac{s}{2R}\right)}{L} \tag{A.3}$$

$$\frac{\partial\theta}{\partial s} = \frac{1}{2L\sqrt{1 - \frac{R^2\sin^2\left(\frac{s}{2R}\right)}{L^2}}} * \frac{\cos\left(\frac{s}{2R}\right)}{2} \tag{A.4}$$

which can be squared and input into the formula, combining common denominators as follows

$$(\delta\theta)^2 = \frac{\left(\sin\frac{s}{2R} - \frac{s\cos\frac{s}{2R}}{2R}\right)^2(\delta R)^2 + \left(\frac{-R\sin\frac{s}{2R}}{L}\right)^2(\delta L)^2 + \left(\frac{\cos\frac{s}{2R}}{2}\right)^2(\delta s)^2}{\left(2L\sqrt{1 - \frac{R^2\sin^2\frac{s}{2R}}{L^2}}\right)^2} \tag{A.5}$$

Taking the square root of both sides and simplifying to get fractional uncertainties gives the final error propagation formula

$$\delta\theta(s,R,L) = \frac{\sqrt{\left(R^2\sin\frac{s}{2R} - \frac{sR\cos\frac{s}{2R}}{2}\right)^2\left(\frac{\delta R}{R}\right)^2 + \left(R^2\sin^2\frac{s}{2R}\right)\left(\frac{\delta L}{L}\right)^2 + \left(\frac{s^2\cos^2\frac{s}{2R}}{4}\right)\left(\frac{\delta s}{s}\right)^2}}{2\sqrt{L^2 - R^2\sin^2\frac{s}{2R}}} \tag{A.6}$$

Assessing the orders-of-magnitude of each fractional uncertainty, we use $\delta R = \delta L = \pm 0.001m$ based on the limitations of the ruler and $\delta s = \pm 0.005m$ from ruler limitations and issues projecting the spherical distance to a measurable flat line, as mentioned in the Error Propagation subsection of the report. Therefore, $\frac{\delta R}{R} = \frac{0.001m}{0.064m} \approx 1.5\%$, $\frac{\delta L}{L} = \frac{0.001m}{0.130m} \approx 0.8\%$. For the arc lengths, we calculate the fractional error for both the inner and outer ring measurements so $\frac{\delta s_{inner}}{s_{inner}} = \frac{0.005m}{0.029} \approx 17.2\%$ and $\frac{\delta s_{outer}}{s_{outer}} = \frac{0.005m}{0.050} \approx 10\%$. Since the magnitude of the uncertainty in the arc length in multiple times greater than the uncertainties in the other measurements, the error propagation formula can simplify to

$$\delta\theta = \frac{s\cos\left(\frac{s}{2R}\right)}{4\sqrt{L^2 - R^2\sin^2\frac{s}{2R}}} * \frac{\delta s}{s} \tag{A.7}$$

as is shown in Equation 5.

B Derivation for Error Propagation in Equation 6

To find the error δd in the interatomic spacing in the graphite, we use the partial derivatives of equation 4 in the general error propagation formula

$$(\delta d)^2 = \left(\frac{\partial d}{\partial V_a}\right)^2(\delta V_a)^2 + \left(\frac{\partial d}{\partial\theta}\right)^2(\delta\theta)^2 \tag{B.1}$$

The two partial derivatives from equation 4 are

$$\frac{\partial d}{\partial V_a} = \frac{h}{2\sin\theta\sqrt{2em_e}} * -\frac{1}{2V_a^{\frac{3}{2}}} \tag{B.2}$$

$$\frac{\partial d}{\partial\theta} = \frac{h}{2\sqrt{2em_eV_a}} * -\csc\theta\cot\theta \tag{B.3}$$

which are squared, input into the general formula, and simplified into the following

$$(\delta d)^2 = \frac{h^2}{8em_eV_a\sin^2\theta} * \left(\frac{1}{4}\left(\frac{\delta V_a}{V_a}\right)^2 + \frac{\cos^2\theta}{\sin^2\theta}(\delta\theta)^2\right) \tag{B.4}$$

Taking the square root of both sides and recognizing that the common term is equal to equation 4 returns

$$\delta d = d\sqrt{\frac{1}{4}\left(\frac{\delta V_a}{V_a}\right)^2 + \left(\frac{\delta\theta}{\tan\theta}\right)^2} \tag{B.5}$$

Since θ is very small, we can approximate $\tan\theta \approx \theta$ such that we get fractional uncertainties in the final error propagation formula

$$\delta d = d\sqrt{\frac{1}{4}\left(\frac{\delta V_a}{V_a}\right)^2 + \left(\frac{\delta\theta}{\theta}\right)^2} \tag{B.6}$$

We can look at the orders-of-magnitude of the uncertainties for the two different spacings we are calculating. In both cases, $\frac{\delta V_a}{V_a} = \frac{150V}{2000V} = 5.0\%$. For the inner ring, $\theta_{10} = 0.06$ rad so $\frac{\delta\theta_{10}}{\theta_{10}} = \frac{0.01}{0.06} = 16.7\%$ and for the outer ring, $\theta_{11} = 0.09$ rad so $\frac{\delta\theta_{11}}{\theta_{11}} = \frac{0.006}{0.09} = 6.7\%$. Since neither uncertainties dominates, Equation 6 is Equation B.6.

C Derivation for Error Propagation in Equation 6

To calculate the error propagation through using the slope to calculate d with equation 7, we first solve the equation for d.

$$d = \frac{h}{2\sqrt{2em_e}} * \frac{1}{c} \tag{C.1}$$

Using the general form for error propagation with partial derivatives gives

$$(\delta d)^2 = \left(\frac{\partial d}{\partial c}\right)^2 (\delta c)^2 \tag{C.2}$$

The partial derivative of d with respect to c is

$$\frac{\partial d}{\partial c} = \frac{h}{2\sqrt{2em_e}} * -\frac{1}{c^2} \tag{C.3}$$

Plugging this in and square rooting and simplifying gives an equation

$$\delta d = \frac{h}{2c\sqrt{2em_e}} * \frac{\delta c}{c} \tag{C.4}$$

Recognizing that the first part of the equation is equal to the estimated d from the slope, we get

$$\delta d = d * \frac{\delta c}{c} \tag{C.5}$$

which is equal to equation 8 above.