Faraday Rotation

Experiment objectives: Observe the *Faraday Effect*, the rotation of a light wave's polarization vector in a material in the presence of a magnetic field directed along the light propagation direction. Determine the relationship between the magnetic field and the rotation angle by measuring the so-called *Verdet constant* of the material. Become acquainted with some new tools: an oscilloscope, a function generator and an amplifier, and a new technique: phase-locking.

Introduction

The term *polarization* refers to the direction of the electrical field in a light wave. Most natural sources of light are not polarized, but we can use a special devices called *polarizers* that transmits electric fields oriented in one direction and absorbs all others. Thus, a well-defined polarization direction, along the polarizer axis. Also, the output of many lasers is polarized.

Imagine a beam of light traveling in the z direction. We then polarize it in the x direction ($\boldsymbol{E} = \hat{\boldsymbol{x}} E_0 \cos(kz - \omega t)$) by passing it through a polarizer and then pass it through a second polarizer, with a transmission axis oriented at an angle θ with respect to the x axis. If we detect the light beam after the second polarizer, the intensity is given by the Malus law:

$$I = I_0 \cos^2 \theta. \tag{1}$$

It is easy to see that light is completely blocked if the two polarizers are oriented at 90^o with respect to each other - we usually refer to such polarizers as being "crossed".

In everyday life most transparent materials do not change the polarization of light. However, in 1845 Michael Faraday discovered that he could rotate the direction of the light polarization using a magnetic field, directed along the light propagation - we now call this phenomena *Faraday effect*. The polarization vector rotates in proportion to the length of the material L, the magnitude of the magnetic field B, and a material dependent constant C_V called the *Verdet constant*:

$$\phi = C_V B L. \tag{2}$$

Typically C_V depends on the wavelength of the light and has a value of a few rad/(T $\cdot$ m). To put it into prospective, the Earth magnetic field is $5 \cdot 10^{-5}$T, and the magnetic field of a strong refrigerator magnet is around 10^{-3}T. Thus, it is easy to estimate that unless very strong magnets are used, the induced polarization rotation is very small, typically well below a milliradian. This is a pretty small angle and it will require a special technique to detect.

The field, created in our laboratory by a solenoid, points in the direction of the light beam, and the magnitude of the magnetic field at the center is . The length of the material, which is a special sort of glass, is long. For a current of 0.1 A, we expect a rotation angle of a few 10^{-4} radians.

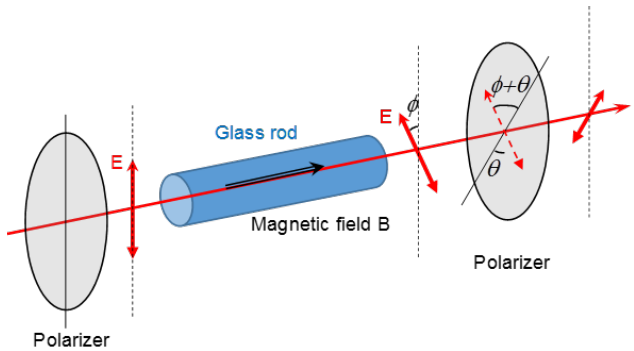

Figure 1: The experimental arrangement for the Faraday rotation measurement. Note that the polarization rotation angle ϕ due to Faraday effect is much smaller than shown.

We are going to take the polarized laser light and direct it through the glass rod, which is inserted into the solenoid. Electric current induces magnetic field in the center of the solenoid at the rate of 11.1 mT/A, that result in the rotation of the laser polarization by a small angle ϕ. The beam then pass through a second polarizer with the transmission axis at an angle θ with respect to the initial polarization of the laser, as shown in Fig.1. The intensity of the light after the second polarizer then depend on the sum of the angle θ and the additional rotation ϕ caused by the magnetic field:

$$\begin{aligned} I &= I_0 \cos^2(\theta+\phi) = I_0(1+\cos(2\theta+2\phi))/2 \\ &= \frac{I_0}{2} + \frac{I_0}{2}\left[\cos 2\theta \cos 2\phi - \sin 2\theta \sin 2\phi\right], \end{aligned} \tag{3}$$

where I_0 is the laser intensity after the first polarizer. As it is mentioned above, the expected polarization rotation angle ϕ is very small, in order of a few 10^{-4} radians. Thus, we can safely assume that $\sin 2\phi \approx 2\phi$ and $\cos 2\phi \approx 1$. Then, the expression for the transmitted laser intensity becomes

$$I = I_0 \frac{1+\cos 2\theta}{2} - \phi I_0 \sin 2\theta. \tag{4}$$

Here the first term describes the laser intensity after the second polarizer at zero magnetic field, and the second term, proportional to $\phi = C_V BL$, shows the change in the transmission due to the Faraday rotation.

To find the value of the Verde constant C_V one obviously needs to find the linear dependence of the polarization rotation angle on the applied magnetic field, according to Eq. 2.

However, this is harder than it may seem. In reality, the term involving ϕ in Eq.(4) is much smaller than the first term, and it can easily be buried in noise from a photo-detector. Thus, to measure this small variation in transmission, we will use an experimental technique, called *phase-locking*.

The phase-locking technique works in the following way. We will vary the magnetic field periodically with time as a sine wave $B(t) = B_0 \sin(2\pi f t)$, causing the light polarization rotation to "wiggle" at the same frequency. Then, if we observe the signal from the photodiode as a function of time, accoreding to Eq.(4) it will look like a large constant with a small sine wave "wobble" on it, along with some random noise with a similar magnitude to the wobble. Oscilloscope can help us to remove the non-time-varying portion of the signal, using a high pass filter (AC coupling). To get rid of the noise, we can take advantage of the fact that we can predict how our expected signal looks in time. Since we know the period and phase of the magnetic field, we can time our observations to be exactly in sync with the magnetic field, and use this to average out the noise. To do that, we start the oscilloscope acquisition at exactly the same time with respect to the magnetic field variation (using triggering function), and average multiple traces. What is left over is the wobble (our useful signal) in I, essentially the $\phi(t) = C_V L B(t) \sin 2\theta$ term. Knowing B, L and θ, we can now determine C_V.

Experimental Setup

The experimental setup is shown in Fig. 2. Its main components are:

- A solenoid to produce the magnetic field. The conversion factor between the magnetic field and the solenoid current is 11.1 mT/A.
- A glass rod inside the solenoid. The length of the rod $L = 10\,\text{cm}$. You'll measure C_V of this material.
- A polarized HeNe laser.
- A second polarizer, the "analyzer".
- A photodiode to measure the laser intensity after the coil and the analyzer.
- A function generator and amplifier to supply current to the solenoid.
- A digital multimeter (DMM) to measure the solenoid current.
- A digital oscilloscope to read out the photodiode voltage.

Experimental Procedure

Preliminaries

Laser & photodiode setup The amplifier includes the laser power supply on the back.

Plug the laser in, **being careful to match colors between the cable and the**

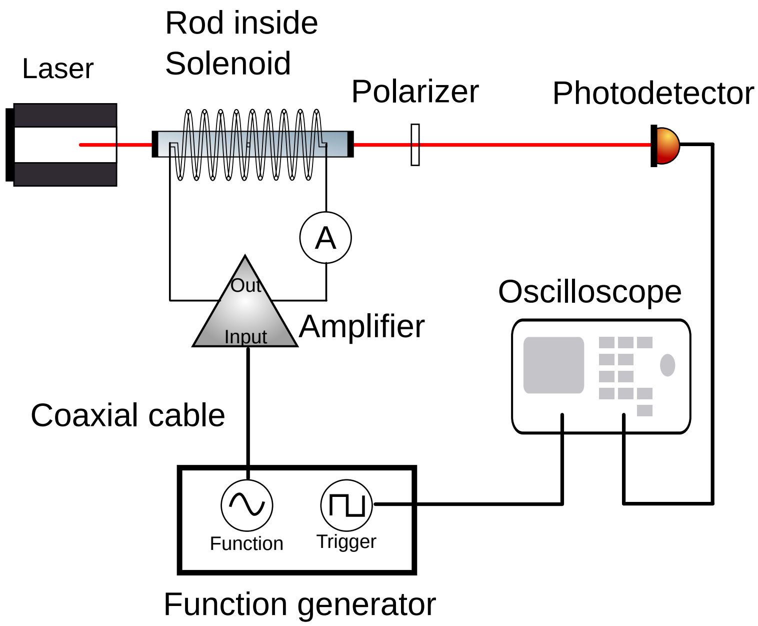

Figure 2: The experimental setup.

power supply's connectors. Align the laser so it travels down the center of the solenoid, through the glass rod, and into the center of the photodiode. Set the photodiode load resistor to $1\,\mathrm{k}\Omega$. Plug the photodiode into a DMM and measure DC voltage.

Calibration: intensity vs. θ We want to understand how the angle between the polarization vector of the laser light and the polarizer direction affects the intensity. Vary the angle of the analyzing polarizer and use a white screen (e.g., piece of paper) to observe how the intensity of the transmitted light changes. Find the angles that give you maximum and minimum transmission. Then, use the DMM to measure the photodiode output as a function of θ, going between the maximum and minimum in small steps (use your first pre-lab exercise to determine the size of the step). Plot the resulting graph and verify that it follows Eq.(1).

Function Generator Setup Plug the function generator output and its trigger (a.k.a. pulse) output into different channels on the scope. Trigger the scope on the trigger/pulse output from the function generator and look at the function generator signal. Modify the function generator to provide a 200 Hz sine wave with an amplitude of about 1 V. You will be using these settings for the rest of the measurements, so do not touch the function generator dials.

Amplifier setup Disconnect the output of the function generator from the oscilloscope and plug it into the amplifier. Use a coaxial↔double-prong connector to feed the amplifier output into the oscilloscope, still triggering it on the generator. Vary the amplifier dial setting and observe the change in the output voltage. At some point, the output will become clipped, as the amplifier reaches its maximum power output. Record this dial setting, and the amplitude of the sine wave, just as the clipping sets in. This is the maximum useful output from the amplifier. Turn the dial all the way down, and hook the amplifier up to the solenoid, with the DMM in series to measure AC current. Go back to the maximum setting and measure the current flowing through the solenoid. This is the maximum useful current. You now have a time varying magnetic field in the solenoid and you can control its magnitude with the amplifier.

Measuring Faraday Rotation

Choice of θ Pick optimal θ, based on your pre-lab exercise, and be sure to tighten the thumbscrew on the polarizer mount to make sure it does not change during the measurements.

Faraday rotation Plug the photodiode output into the scope, set the scope so its channel is DC coupled, and make sure that the "probe" setting is at 1x. Turn the amplifier dial about halfway to the maximum setting you found. Observe the photodiode trace on the scope, perhaps changing the volts/div setting so you can see the trace more clearly. What is the voltage? Record it. The changing magnetic field should be causing a change in the polarization angle of the laser light, which should cause a sinusoidal time dependence to the photodiode signal, referred to as the "wobble". Check if you can see any wobble.

AC coupling The wobble is riding atop a large constant (DC) signal. The scope can remove the DC signal by "AC coupling" the photodiode channel. This essentially directs the scope input through a high pass filter. Do this, and then set the photodiode channel to the 2 mV setting. You should now see a wobble. Vary the amplifier dial setting and notice how the amplitude of the wobble changes. You are seeing the Faraday effect.

Reduce the noise The signal is noisy, but now we'll really benefit from knowing the waveform that the function generator is producing. Because we trigger the scope on the function generator, the maxima and minima for the useful signal will occur at the same point on the scope screen (and in its memory bank). The scope has a feature that allows you to average multiple triggers. Doing this mitigates the noise, since at each point on the trace we are taking a mean, and the uncertainty in a mean decreases as we increase the number of measurements N as $1/\sqrt{N}$. Turn on the averaging feature by going to the "Acquire" menu. Observe how the averaged trace becomes more stable as you increase the number of traces being averaged. The larger the better, but ~100 traces should be enough.

Take measurements You should now systematically measure the amplitude of the wobble as a function of the current in the solenoid. There should be a linear relationship, which can be fit to extract C_V. Take about 10 measurements, evenly separated between the smallest current for which there is a measurable wobble, and the maximum you found earlier. In each case, you want to start acquisition (Run/Stop on the scope), let the averaged signal converge onto a nice sine wave, stop acquisition and measure the amplitude of the signal using the scope's cursors. One measurement is shown in Fig. 3. Record the negative and positive amplitudes V_{low} and V_{high} ($\pm 640\,\mu$V in Fig. 3) and the peak to peak voltage ΔV (1.28 mV), along with the current in the coil – I_{coil} . Estimate the uncertainty in your measurements based on the noise in your averaged signal and/or the achievable accuracy of oscilloscope.

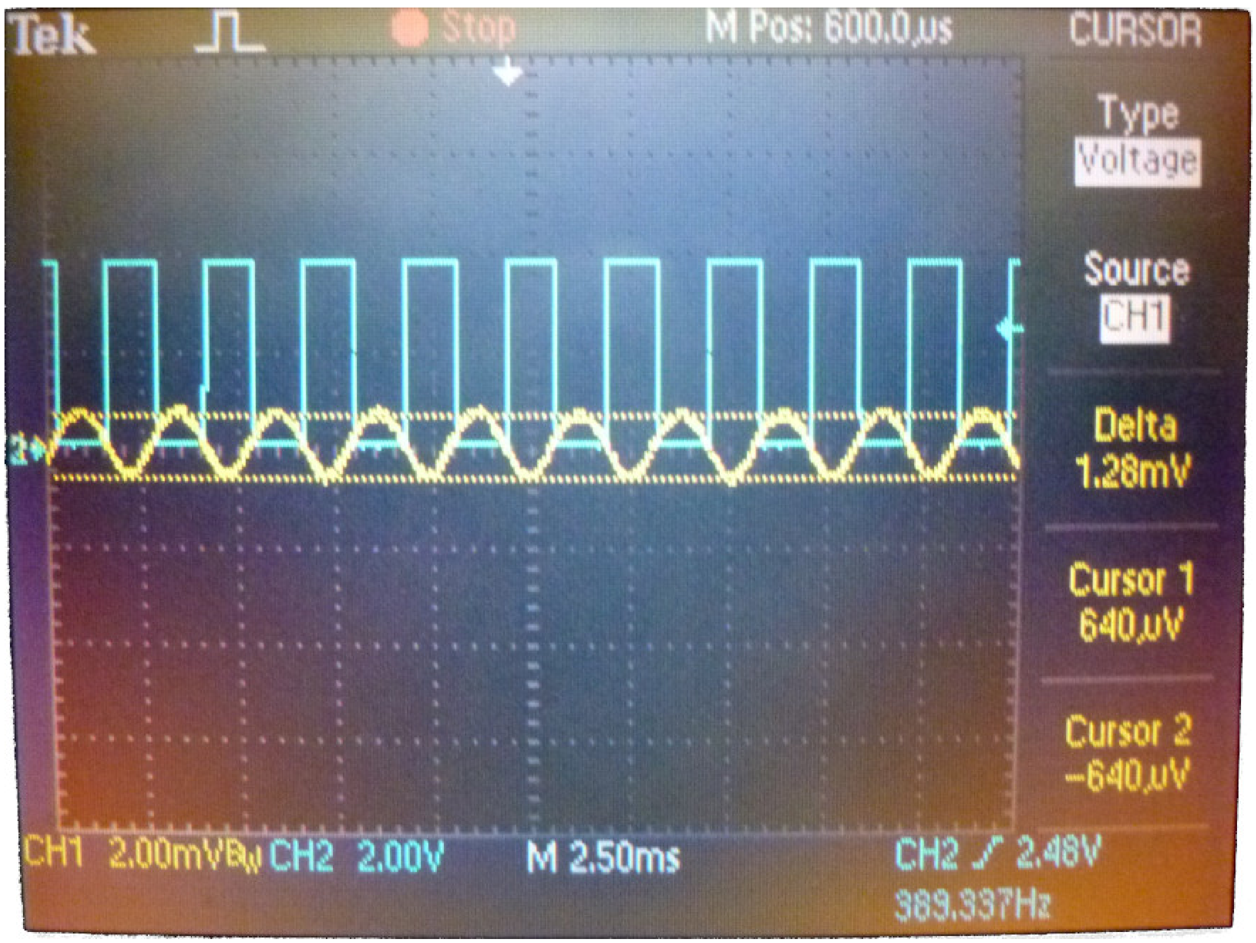

Figure 3: An example scope trace. The yellow curve is the output of the photodiode, AC coupled and averaged over 128 traces. The blue curve is the trigger output from the function generator and is being used to trigger the scope readout so that it's in phase with the changing magnetic field. The maximum and minimum amplitudes of the photodiode signal are measured with the scope's cursors.

Data Analysis

A variation in the angle ϕ is related to a variation in the magnetic field B according to:

$$\Delta\phi = C_V L \Delta B \tag{5}$$

We want to use this equation to extract C_V. Both ϕ and B vary with time as sine waves. We'll take $\Delta\phi$ and ΔB to be the amplitude of those waves (half the peak to peak). We didn't directly measure either quantity, but we can compute them. For ΔB it's easy: $\Delta B = 11.1\,\mathrm{mT/A} \times \sqrt{2} I_{\mathrm{coil}}$, where the $\sqrt{2}$ accounts for the fact that DMMs measure the root-mean-squared (RMS) value of an AC signal, not the peak value.

Figuring out $\Delta\phi$ is a little more difficult. We need to use our calibration dataset, which relates the photodiode signal to the angle between the laser polarization and the analyzer's orientation. From that dataset, you can estimate $\mathrm{d}V/\mathrm{d}\theta$ at the value of θ you are using in your experiment. Of course a change in the polarization of the laser, $\Delta\phi$ is equivalent to holding the laser polarization constant and changing the angle of the analyzer by $-\Delta\theta$. So, we can calculate:

$$\Delta\phi = \left[\frac{\mathrm{d}V}{\mathrm{d}\theta}\right]^{-1} \frac{\Delta V}{2} \tag{6}$$

The factor of two is because we defined ΔV as the peak to peak voltage.

Complete the analysis by fitting $\Delta\phi$ vs. ΔB to a straight line and then use Eq. 5 to extract C_V and its uncertainty (consider what the intercept should be). Check the quality of your linear fit. If there are visible deviations, discuss possible reasons.