PHYS 401: Electricity & Magnetism 1
Tuesday, May 4, 2021

Formula Study Sheet

(i.e. formulas that you should know)

Gradient theorem

$$\int_{\vec{r}_a}^{\vec{r}_b} (\vec{\nabla} f) \cdot d\vec{l} = f(\vec{r}_b) - f(\vec{r}_a)$$

path P

Divergence Theorem

$$\int_V (\vec{\nabla} \cdot \vec{F})\, d^3r = \oint_{S(V)} \vec{F} \cdot d\vec{s}$$

Stokes's Theorem

$$\int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = \oint_{C(S)} \vec{F} \cdot d\vec{\ell}$$

Divergence of $1/r^2$ - point source

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} = 4\pi\delta^3(\vec{r}) \quad \& \quad \nabla^2 \frac{1}{r} = -4\pi\delta^3(\vec{r})$$

Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

Electric field of a point charge q at $\vec{r}'$

$$\vec{E}_q = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|^2} (\widehat{r - r'})$$

Electric field of a charge distribution $\rho(\vec{r})$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^2} (\widehat{r - r'}) d^3r'$$

Potential of a point charge q at $\vec{r}'$

$$V_q(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|}$$

Potential of a charge distribution $\rho(\vec{r})$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$$

Electric field and potential

$$\vec{E} = -\vec{\nabla} V \quad \& \quad V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$$

Electric field of a plane of charge

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

Electric field across a plane of charge

$$\Delta E_\perp = \frac{\sigma}{\epsilon_0} \quad \& \quad \Delta E_\parallel = 0$$

Gauss's law

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0} \quad \& \quad \oint_S \vec{E} \cdot d\vec{s} = \frac{q_{enclosed}}{\epsilon_0}$$

Electrostatic field has no curl

$$\vec{\nabla} \times \vec{E} = 0$$

Laplace's equation

$$\nabla^2 V(\vec{r}) = 0$$

Poisson's equation

$$\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

Electrostatic energy

$$U_E = \frac{\epsilon_0}{2} \int \vec{E}^2\, d^3r$$

Capacitor of capacitance C

$$C = \frac{Q}{V} \quad \& \quad U_E = \frac{1}{2} C V^2$$

Fourier basis orthogonality relation

$$\int_0^\pi \sin(mx)\sin(nx)\, dx = \frac{\pi}{2}\delta_{mn}$$

Legendre basis orthogonality relation

$$\int_{-1}^{1} \mathrm{P}_k(x)\,\mathrm{P}_l(x)\,dx = \frac{2}{2k+1}\delta_{kl}$$

Separation of variables: general solution forms for spherical symmetry

$$V(r,\theta) = \sum_{n=0}^{\infty}\left(A_n r^n + B_n r^{-(n+1)}\right)P_n(\cos\theta)$$

$$V(r,\theta) = \begin{cases} \sum_{n=0}^{\infty} C_n \left(\frac{r}{R}\right)^n P_n(\cos\theta) & \text{for } r \le R \\ \sum_{n=0}^{\infty} C_n \left(\frac{R}{r}\right)^{n+1} P_n(\cos\theta) & \text{for } r \ge R \end{cases}$$

Potential of an electric dipole

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Electric dipole moment

$\vec{p} = \sum_{i=1}^{N} q_i \vec{r}_i$ & $\vec{p} = \int_V \rho(\vec{r}')\vec{r}'d^3r'$

$\vec{p} = q\vec{d}$ ($\vec{d}$ points from $-q$ to $+q$)

Torque, force, and energy for an electric dipole

$\vec{\tau} = \vec{p}\times\vec{E}$, $\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E}$, $U_{dipole} = -\vec{p}\cdot\vec{E}$

Bound charge and polarization

$\rho_b(\vec{r}) = -\vec{\nabla}\cdot\vec{P}(\vec{r})$ and $\sigma_b(\vec{r}) = \vec{P}(\vec{r})\cdot\hat{n}$

Electric displacement field: $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

"Gauss's law" for electric displacement field

$\vec{\nabla}\cdot\vec{D} = \rho_{free}$ & $\oint_S \vec{D}\cdot d\vec{s} = q_{free,enclosed}$

Boundary conditions for dielectrics

$\left(\vec{D_1} - \vec{D_2}\right)_\perp = \sigma_{free}$

$\left(\vec{D_1} - \vec{D_2}\right)_\parallel = \left(\vec{P_1} - \vec{P_2}\right)_\parallel$

Linear dielectrics

$\vec{D} = \epsilon\vec{E}$ with $\epsilon = \epsilon_0(1+\chi_e)$

$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{free}}{\epsilon}$

$\epsilon_1\frac{\partial V}{\partial n} - \epsilon_2\frac{\partial V}{\partial n} = \sigma_{free}$ $\hat{n}$ points from 2 to 1

Lorentz force law: $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$

Force on a line current: $\vec{F} = I\int d\vec{l}\times\vec{B}$

Current density & continuity equation

$\vec{J} = \rho\vec{v}$ & $\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$

Biot-Savart law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}')\times(\widehat{r - r'})}{|\vec{r} - \vec{r}'|^2}d^3r'$$

Ampère's law

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ & $\oint_{loop}\vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$

No magnetic monopoles law: $\vec{\nabla}\cdot\vec{B} = 0$

Magnetic vector potential: $\vec{B} = \vec{\nabla}\times\vec{A}$

Coulomb gauge definition: $\vec{\nabla}\cdot\vec{A} = 0$

Vector potential in Coulomb gauge

$\nabla^2\vec{A} = -\mu_0\vec{J}$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|}d^3r'$$

Magnetic dipole potential (vector)

$$\vec{A}_{dipole}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

Magnetic moment (current I, area $\vec{a}$): $\vec{m} = I\vec{a}$

Torque, force, and energy for a magnetic dipole

$\vec{\tau} = \vec{m}\times\vec{B}$, $\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}\right)$, $U_{dipole} = -\vec{m}\cdot\vec{B}$

Bound current and magnetization

$\vec{J}_b = \vec{\nabla}\times\vec{M}$ and $\vec{K}_b = \vec{M}\times\hat{n}$

Auxiliary field: $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

"Ampère's law" for the auxiliary field

$\vec{\nabla}\times\vec{H} = \vec{J}_{free}$ & $\oint_{loop}\vec{H}\cdot d\vec{l} = I_{free,enclosed}$

Linear magnetic material

$\vec{M} = \chi_m\vec{H}$ & $\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu\vec{H}$