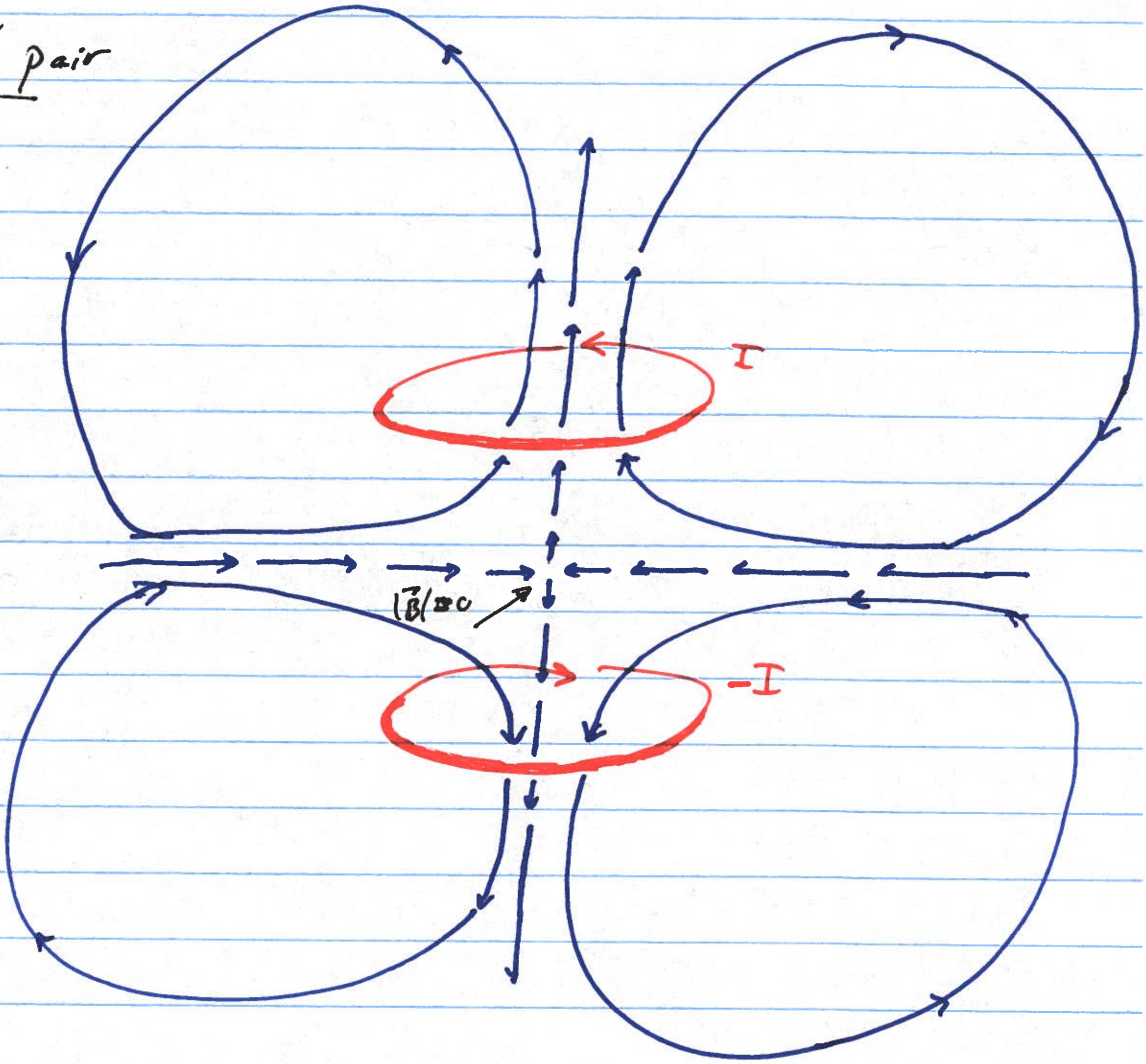
Monday, May 3, 2021

Important magnet coil structures (continued)

Anti-Helmholtz coil pair

quadrupole magnetic field



near origin (center), $|\vec{B}|$ is linear. In fact, $\vec{B} = -\beta x \hat{x} - \beta y \hat{y} + 2\beta z \hat{z}$

[$2\beta \equiv$ gradient along $\hat{z} \equiv [T]/[m]$] (with $\vec{\nabla} \cdot \vec{B} = 0$)

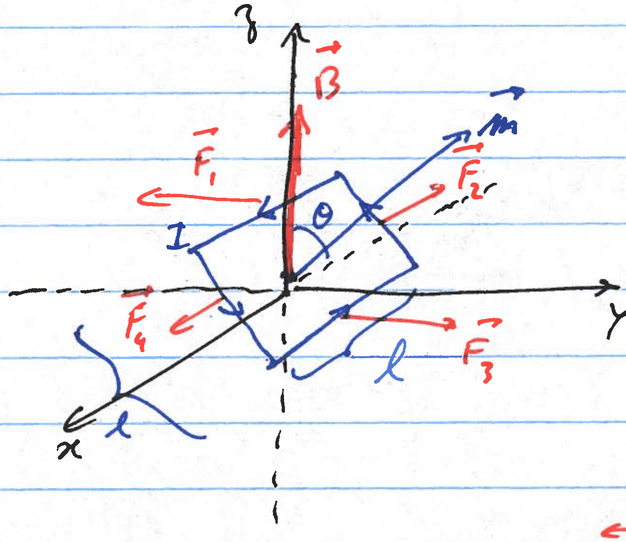
note: An anti-H coil pair produces a $|\vec{B}|$ minimum, and it can be used to trap "weak field seeking spin states"

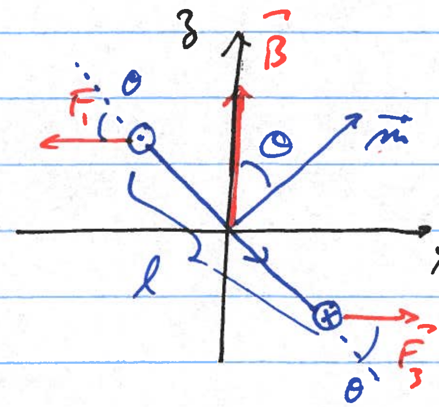
magnetic moments

Forces on magnetic dipoles [chapter 6]

Torque

$$\vec{F} = q\vec{v}\times\vec{B}$$
$$= \vec{I}\times\vec{B}\,l$$





The net force is zero, but there
$(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0)$
is a net torque from F_1 & F_3

$\Rightarrow$ $\boxed{\text{Torque tends to align } \vec{m} \text{ with } \vec{B}}$

$$\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$
$$= \frac{l}{2} I B l \sin\theta\,\hat{x} + \frac{l}{2} I B l \sin\theta\,\hat{x} = \underbrace{I \overset{\text{area}}{l^2}}_{m} B \sin\theta\,\hat{x}$$

$$= m B \sin\theta\,\hat{x}$$
$$= \vec{m}\times\vec{B}$$

more generally $\boxed{\vec{\tau}_{dipole} = \vec{m}\times B}$ = torque on magnetic dipole

Force

The force on a magnetic moment in a constant B-field is zero

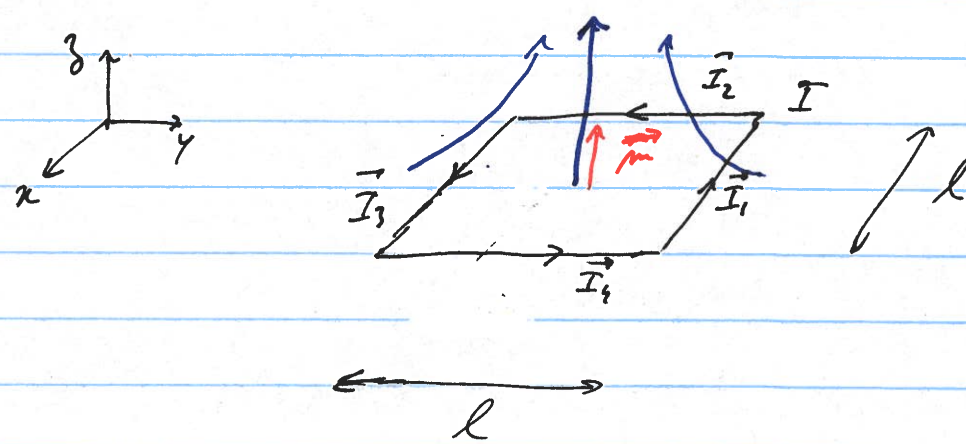
$$\vec{F} = I \oint (d\vec{\ell} \times \vec{B}) = I \underbrace{\left(\oint d\vec{\ell} \right)}_{\substack{=0 \\ \text{(closed loop)}}} \times \vec{B} = 0$$

The force on a magnetic moment in a magnetic field with a gradient is not zero. In fact,

$$\boxed{\vec{F} = \nabla(\vec{m} \cdot \vec{B})}$$

example: magnetic dipole moment near center of anti-H coil

$m = I\ell^2 \hat{z}$



$$\vec{B} \simeq -\beta x \hat{x} - \beta y \hat{y} + 2\beta z \hat{z}$$

$$\vec{F}_1 = \vec{I}_1 \times \vec{B}\,\ell = -I\hat{x} \times \left(-\beta x \hat{x} - \beta y \hat{y} + 2\beta z \hat{z} \right) \ell$$

$y_1 = +\ell/2$

$$= -I\ell \left(0 - \beta y_1 \hat{z} + 2\beta z(-\hat{y}) \right)$$

$$\Rightarrow \quad \vec{F}_1 = \beta I \frac{\ell^2}{2} \hat{z} + 2\beta I \ell z \hat{y}$$

Similarly, $\vec{F}_3 = \vec{I}_3 \times \vec{B} \ell$

$$= I\hat{x}\left(-\beta x \hat{x} - \beta y \hat{y} + 2\beta z \hat{z}\right)\ell$$

$$= I\ell\left(0 - \beta y_3 \hat{z} + 2\beta z(-\hat{y})\right)$$

$y_3 = -\ell/2$

$$= \beta I \frac{\ell^2}{2} \hat{z} - 2\beta I \ell z \hat{y}$$

$$\Rightarrow \quad \vec{F}_1 + \vec{F}_3 = \beta I \frac{\ell^2}{2}\hat{z} + \cancel{2\beta I \ell z \hat{y}} + \beta I \frac{\ell^2}{2}\hat{z} - \cancel{2\beta I \ell z \hat{y}}$$

$$= \beta I \ell^2 \hat{z}$$

Similarly $\vec{F}_2 + \vec{F}_4 = \beta I \ell^2 \hat{z}$

Thus $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \underbrace{2\beta}_{\vec{\nabla}|\vec{B}| = 2\beta\hat{z}} \underbrace{I\ell^2}_{|\vec{m}|} \hat{z}$

We can see that $\vec{F}_{total} = \vec{\nabla}(\vec{m} \cdot \vec{B})$

note: $U_{\substack{potential \\ energy}} = -\vec{m} \cdot \vec{B} = H_{Zeeman}$

Matching conditions for $\vec{B}$ across a surface current

Electrostatics

$$\vec{E}_{\substack{\text{surface}\\\text{of charge}}} = \frac{\sigma_s}{2\varepsilon_0}\hat{n}$$ (no external E-fields)

External E-fields can be present

$$\Delta\vec{E}\Big|_{\text{surface}} = \vec{E}_1\Big|_s - \vec{E}_2\Big|_s = \frac{\sigma_s}{\varepsilon_0}\hat{n}_1$$

$$\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$$

$$\Delta\vec{E}\Big|_s \cdot \hat{n}_1 = \frac{\sigma_s}{\varepsilon_0}$$

Qualitatively:

→ Normal/perpendicular component is affected

→ parallel component is unaffected (i.e. is continuous)

V is continuous across boundary

Magnetostatics

$$\vec{B}_{\substack{\text{surface}\\\text{current}}} = \frac{\mu_0}{2}\vec{K}\times\hat{n}$$ (no ext. B-fields)

$$\Delta\vec{B} = \vec{B}_1 - \vec{B}_2 = \mu_0 \vec{K}\times\hat{n}_1$$

$$B_{1,\parallel} - B_{2,\parallel} = \mu_0 K$$

from Ampère's Law

"$\parallel$" → { $\parallel$ to surface, $\perp$ to $\vec{K}$ }

consistent

$$\Delta\vec{B}\Big|_s \cdot \hat{n}_1 = 0 \quad (\text{from } \vec{\nabla}\cdot\vec{B} = 0)$$

→ perpendicular component is unaffected (i.e. is continuous)

→ parallel component to surface is affected (but not the one parallel to $\vec{K}$)

$\vec{A}_\parallel$ is continuous (from Ampère's law)

$\vec{A}_\perp$ is continuous in Coulomb gauge.

note: $\frac{\partial A}{\partial n}$ is discontinuous.

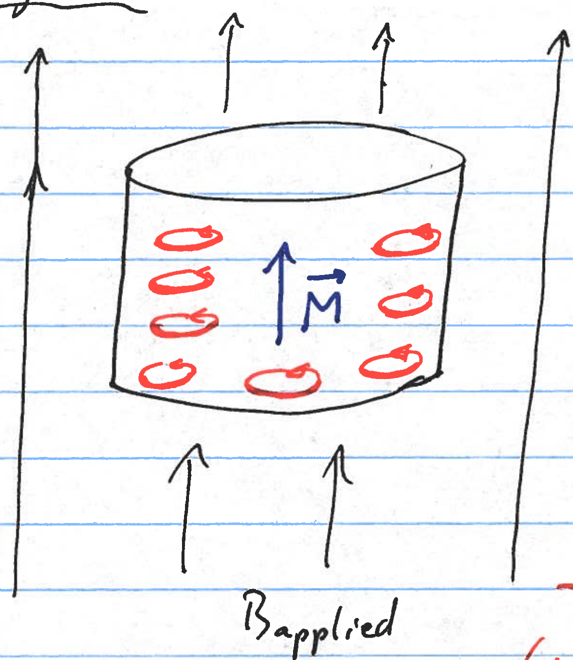
B-fields in Matter [chapter 6]

Basics of magnetism

$\vec{M}$ = "magnetization"

= magnetic moment per unit volume

generally from e^- spins, but it can be useful to think of small current loops (classical description)



Paramagnet

→ attracted to high $\vec{B}$

→ "high field seeker"

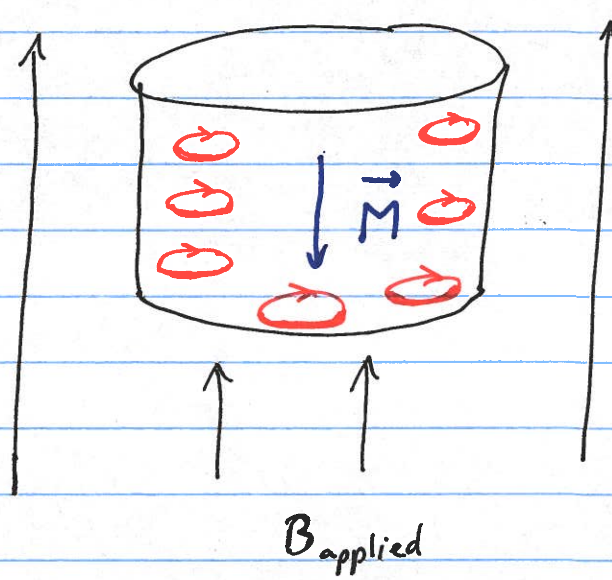
→ $\vec{M} \parallel \vec{B}_{applied, local}$

(↳ similar to dielectrics $\vec{P} \parallel \vec{E}_{applied, local}$)

Diamagnet

→ repelled from high $\vec{B}$

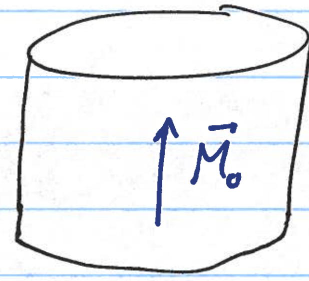
→ "low-field seeker"



Ferromagnet

→ $\vec{M}$ depends on history of $\vec{B}_{applied}$.

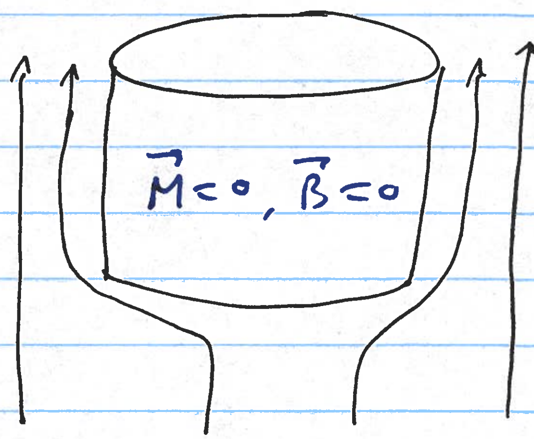
→ Paramagnet with memory



$\vec{B}_{applied} = 0$

Superconductor

→ sort of like a super/perfect diamagnet.



$\vec{B}_{applied}$