PHYS 401: Electricity & Magnetism 1 Thursday, March 11, 2021

Formula Study Sheet

(i.e. formulas that you should know)

Gradient theorem

$$\int_{\vec{r}_a}^{\vec{r}_b} (\vec{\nabla}f) \cdot d\vec{l} = f(\vec{r}_b) - f(\vec{r}_a)$$
path P

Divergence Theorem

$$\int_{V} (\vec{\nabla} \cdot \vec{F}) d^{3}r = \oint_{S(V)} \vec{F} \cdot d\vec{s}$$

Stokes's Theorem

$$\int_{S} (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = \oint_{C(S)} \vec{F} \cdot d\vec{\ell}$$

Divergence of $1/r^2$ - point source $\vec{\nabla} \cdot \frac{\dot{r}}{r^2} = 4\pi \delta^3(\vec{r}) \quad \& \quad \nabla^2 \frac{1}{r} = -4\pi \delta^3(\vec{r})$

Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

Electric field of a point charge q at \vec{r}'

$$\vec{E}_q = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|^2} (\hat{r - r'})$$

Electric field of a charge distribution $\rho(\vec{r})$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^2} (\hat{r - r'}) d^3r'$$

Potential of a point charge q at \vec{r}'

$$V_q(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|}$$

Potential of a charge distribution $\rho(\vec{r})$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{V} \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$$

Electric field and potential $\vec{E} = -\vec{\nabla}V \& V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$

Electric field of a plane of charge $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$

Electric field across a plane of charge $\Delta E_{\perp} = \frac{\sigma}{\epsilon_0} \& \Delta E_{\parallel} = 0$

Gauss's law

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0} \& \oint_S \vec{E} \cdot d\vec{s} = \frac{q_{enclosed}}{\epsilon_0}$$

Electrostatic field has no curl $\vec{\nabla} \times \vec{E} = 0$

Laplace's equation $\nabla^2 V(\vec{r}) = 0$

Poisson's equation

$$\nabla^2 V(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0}$$

Electrostatic energy

$$U_E = \frac{\epsilon_0}{2} \int \vec{E}^2 \, d^3 r$$

Capacitor of capacitance C $C = \frac{Q}{V} \& U_E = \frac{1}{2}CV^2$

Fourier basis orthogonality relation $\int_0^{\pi} \sin(mx) \sin(nx) \, dx = \frac{\pi}{2} \delta_{mn}$