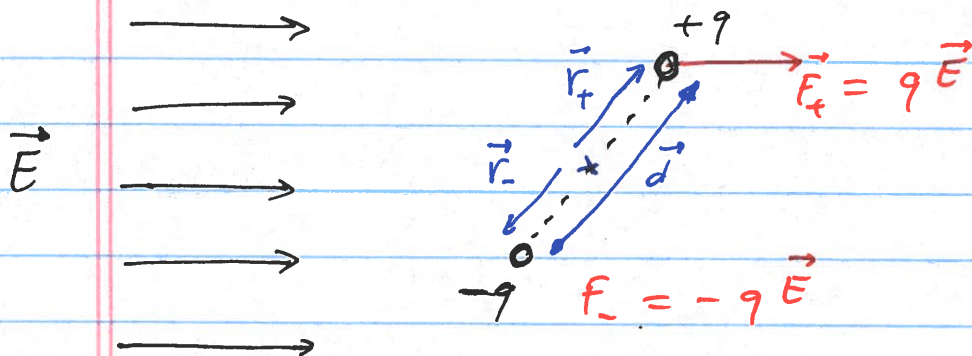


Monday, March 29, 2021

## Dipole forces [chpt 4.1.3]

Torque on a dipole in a uniform E-field



note: net force  
 $\vec{F}_{\text{total}} = \vec{F}_+ + \vec{F}_-$   
 $= 0$

$$\vec{\tau} = \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_-$$

$$= \frac{\vec{d}}{2} \times q\vec{E} + \left(-\frac{\vec{d}}{2}\right) \times (-q\vec{E})$$

$$= \vec{d} \times q\vec{E} = q \underbrace{\vec{d}}_{\vec{p}} \times \vec{E}$$

$$\Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}} = \text{torque on a dipole}$$

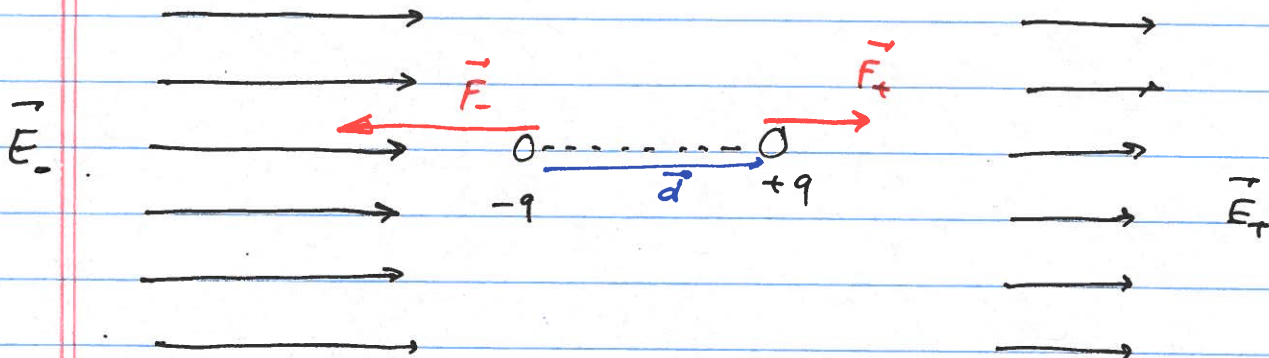
$\vec{N}$  in Griffith book

note: torque is zero  $\vec{p} \parallel \vec{E}$  (parallel or anti-parallel)  
stable unstable

↳ a molecule with a permanent dipole moment will align with the electric field.

↑  
 i.e. a polar molecule,  
 e.g. water

## Force on a dipole in a non-uniform E-field



$$\begin{aligned}\vec{F}_{\text{total}} &= \vec{F}_+ + \vec{F}_- \\ &= q\vec{E}_+ + (-q)\vec{E}_- = q(\vec{E}_+ - \vec{E}_-)\end{aligned}$$

$$\Delta \vec{E} \quad d_x, d_y, d_z \text{ are small}$$

$$\Delta \vec{E} = \left( \frac{\partial E_x}{\partial x} d_x + \frac{\partial E_x}{\partial y} d_y + \frac{\partial E_x}{\partial z} d_z, \dots \right)$$

$$= \left( (\vec{d} \cdot \vec{\nabla}) E_x, (\vec{d} \cdot \vec{\nabla}) E_y, (\vec{d} \cdot \vec{\nabla}) E_z \right)$$

$$= (\vec{d} \cdot \vec{\nabla}) \vec{E}$$

$$= q(\vec{d} \cdot \vec{\nabla}) \vec{E} = (\vec{p} \cdot \vec{\nabla}) \vec{E}$$

$$\Rightarrow \boxed{\vec{F} = (\vec{p} \cdot \vec{\nabla}) \vec{E}} = \text{force on a dipole in a non-uniform E-field}$$

note: if  $\vec{p} = \text{cst}$  (i.e. does not change orientation or magnitude)  
then  $\vec{F} = \vec{\nabla}(\vec{p} \cdot \vec{E})$

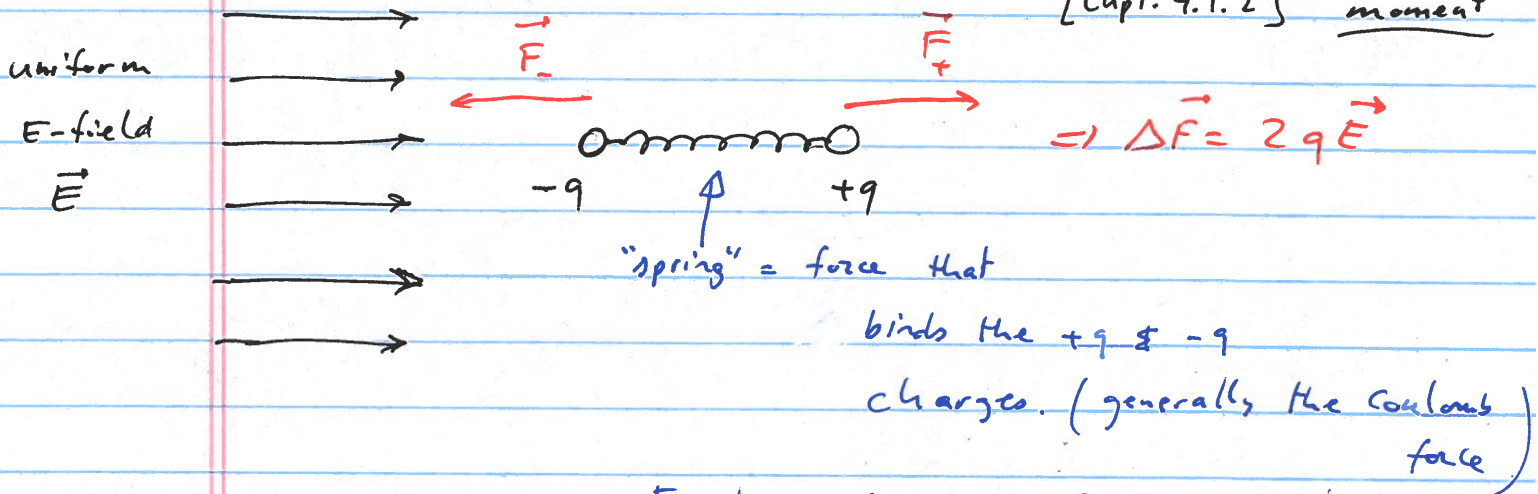
the potential energy of the dipole is then such that

$$\vec{F}_{\text{dipole}} = -\vec{\nabla} U_{\text{dipole}}$$

$$\Rightarrow U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$$

Internal force on a dipole, atom, molecule: Induced dipole

[chpt. 4.1.2] moment



internal "spring" force:  $\vec{F} = -k \frac{\vec{p} \cdot \vec{d}}{d^3}$

gives rise to a dipole moment  $q \cdot d$   
 $q \cdot \vec{d}$

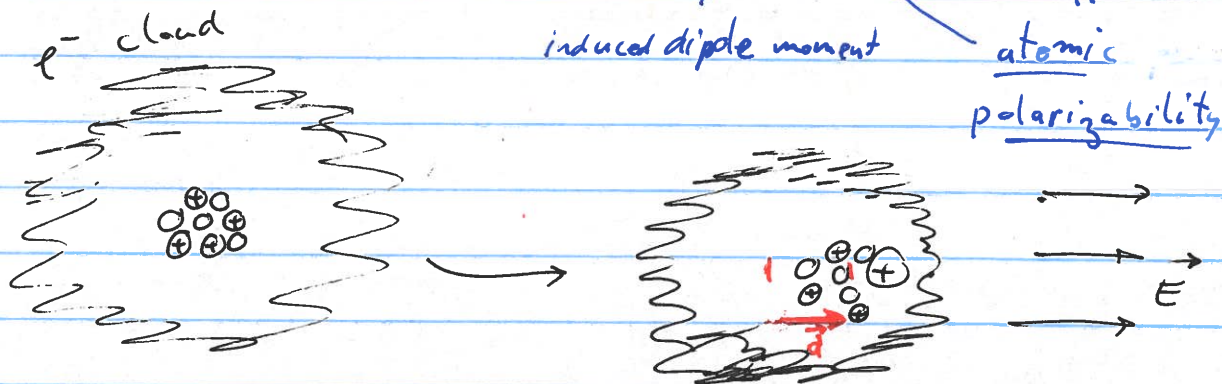
Typically, for atoms & molecules,

$$\vec{p} = \alpha \vec{E}$$

applied E-field

induced dipole moment

atomic polarizability.



$\alpha = \text{DC polarizability} > 0$  [see Stark Effect in QM]

back to dipole force

$$\begin{aligned}
 \vec{F} &= (\vec{p} \cdot \vec{\nabla}) \vec{E} = (\alpha \vec{E} \cdot \vec{\nabla}) \vec{E} && \text{if locally } \vec{E} = E \hat{z} \\
 &= \alpha E \frac{\partial E}{\partial z} \\
 &= \frac{\partial}{\partial z} \left( \frac{1}{2} \alpha E^2 \right) \\
 \Rightarrow \vec{F} &= \vec{\nabla} \left( \frac{1}{2} \alpha E^2 \right)
 \end{aligned}$$

$\Rightarrow$  potential energy of an atom in an E-field:

$$U_{\text{atom}} = -\frac{1}{2} \alpha E^2$$

$\Rightarrow$  Energy of atom becomes more negative as the E-field increases

$\hookrightarrow$  atoms are attracted to regions of high E-field (and molecules)

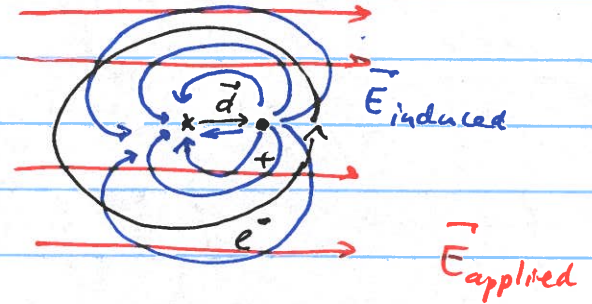
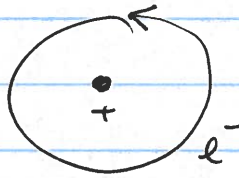
$\hookrightarrow$  atoms are high-field seekers (electric field) and molecules

Example: For an IR laser, far from any atomic resonances, the laser is quasistatic to an atom ( $e^-$  dynamics/response are much faster than  $f_{\text{laser}} = \text{laser frequency}$ ).

$\hookrightarrow$  atoms are attracted to laser focus, where  $E_{\text{laser}}$  is (i.e. high intensity) highest

## Electrostatics in Matter

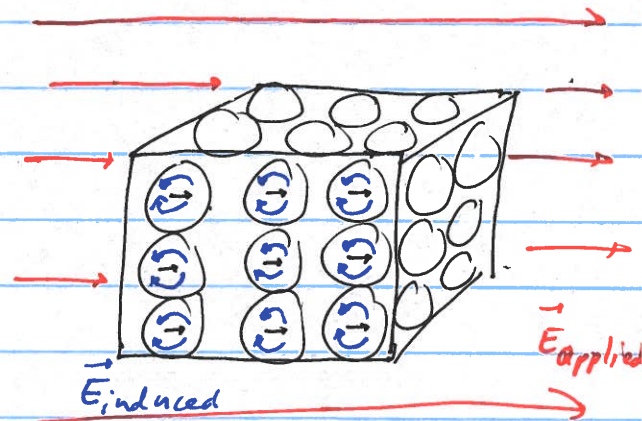
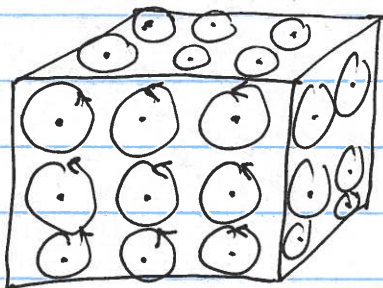
1) Consider a classical atom in an  $E$ -field



$$\vec{P}_{\text{induced}} \propto \vec{E}$$

→ the induced dipole  $\vec{P}$  produces its own electric field,  $\vec{E}_{\text{induced}}$ , which partially counteracts the applied  $E$ -field  $\vec{E}_{\text{applied}}$ .

2) Consider a block of matter made up of similar-behaving atoms



⇒ you expect to ~~polarize~~ polarize the material and so there should be an associated  $\vec{E}_{\text{induced}}$ .

↳ dipole moment per unit volume:  $\vec{P} \stackrel{?}{=} \propto \vec{E}$

we'll come back to this later

Thus  $\vec{E}_{\text{total}}$  is given by  $\vec{E}_{\text{total}} = \vec{E}_{\text{applied}} + \underbrace{\vec{E}_{\text{induced}}}_{E_{\text{self}} \text{ or } E_{\text{internal}}}$

→ you expect the charges in the material to ~~not~~ not exactly cancel (i.e. you get local dipoles.)

↳ volume bound charge  $\rho_b$   
 ↳ surface bound charge  $\sigma_b$

↳ total charge density:  $\rho_{\text{total}}(\vec{r}) = \rho_{\text{applied}}(\vec{r}) + \rho_b(\vec{r}) + \sigma_b(\vec{r})$

$\rho_{\text{free}} = \text{free charge}$   
 = charge put there by the experimentalist

We also require  $\int_{V \text{ of material}} \rho_b(\vec{r}) d^3r + \int_{S \text{ of material}} \sigma_b(\vec{r}) ds = 0 = \text{total bound charge}$

definition:  $\vec{P} \equiv$  dipole moment per unit volume

↳ we will model ~~poly~~ polarized matter as a collection of dipoles (no quadrupoles)

Q: What is the relationship between  $\vec{P}$ ,  $\rho_b$ , and  $\sigma_b$ ?

A: let's derive it!