PHYS 401: Electricity & Magnetism I
Due date: Wednesday, February 8, 2023

Problem set #2

1) Griffiths (4th Ed.) problem 1.21
Prove the following product rules:

(a) $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$

(b) $\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) = \vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)$

(c) $\vec{\nabla}\times\left(f\vec{A}\right) = f\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\times(\vec{\nabla}f)$

2) Griffiths (4th Ed.) problem 1.36
(a) Show that

$$\int_S f\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{s} = \int_S \left(\vec{A}\times(\vec{\nabla}f)\right)\cdot d\vec{s} + \oint_P f\vec{A}\cdot d\vec{l}$$

(b) Show that

$$\int_V \vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right)d\tau = \int_V \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)d\tau + \oint_S \vec{A}\times\vec{B}\cdot d\vec{s}$$

3) Griffiths (4th Ed.) problem 1.40
Compute the divergence of the function

$$\vec{v} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$$

Check the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin (e.g. Fig. 1.40).

4) Variation on the Divergence Theorem
Prove the following two integral theorems:

a) $\int_V \left(\vec{\nabla}\times\vec{F}\right)d^3r = -\int_S \vec{F}\times d\vec{s}$

b) $\int_V \vec{\nabla}f d^3r = \int_S f d\vec{s}$

Where f is a scalar function, $\vec{F}$ is a vector function, and S is the bounding surface for a volume V.
Hint: You may want to consider "multiplying" the appropriate field by a constant vector field.

5) Variation on Stokes' theorem

Prove the following integral theorem: $\int_S \hat{n} \times \vec{\nabla} f \, dS = \int_C f \, \overrightarrow{dl}$

Where f is a scalar function, S is surface with contour C, $\hat{n}$ is a unit vector locally perpendicular to S, and $\overrightarrow{dl}$ is a differential line element along C.

6. Green's identities

a) Use the divergence theorem with $\vec{A} = \phi \vec{\nabla} \psi$ to prove *Green's first identity*:

$$\int_V [\phi \vec{\nabla}^2 \psi + \vec{\nabla}\phi \cdot \vec{\nabla}\psi] \, d^3r = \int_S \phi \vec{\nabla}\psi \cdot \overrightarrow{dS}$$

$\phi(\vec{r})$ and $\psi(\vec{r})$ are arbitrary (well-behaved) scalar functions, and *V* is a volume with surface *S*.

b) Prove *Green's second identity*:

$$\int_V [\phi \vec{\nabla}^2 \psi - \psi \vec{\nabla}^2 \phi] \, d^3r = \int_S [\phi \vec{\nabla}\psi - \psi \vec{\nabla}\phi] \cdot \overrightarrow{dS}$$