PHYS 401: Electricity & Magnetism I
Due date: Wednesday, April 12, 2023

Problem set #9

1) Problem 4.14: Total bound charge

2) Problem 4.16: Cavities in a dielectric
Hints: (a) Consider the solution to the opposite problem of a uniformly polarized sphere in example 4.2. (b) Figure out where the surface bound charge σ_b is located and determine whether it has an effect in this needle geometry. (c) Figure out where the surface bound charge σ_b is located and determine its effect on the field(s) in the cavity.

3) Problem 4.36: Snell's law for electric fields

4) Dielectric sphere with free charge

A dielectric sphere (with dielectric constant κ) of radius R is filled with a uniform free charge density ρ_c.

a) Find the polarization $\vec{P}(\vec{r})$.

b) Calculate the total bound charge of the sphere (volume and surface). Explain the result briefly.

5) Dielectric sphere in a uniform electric field
(Note: This problem is based on Example 4.7 in the book by Griffiths, p. 193-194)

In this problem, you will calculate the potential $V(r,\theta)$ for a full dielectric sphere of radius R and dielectric constant $\epsilon_r = \epsilon/\epsilon_0$ placed in an otherwise uniform electric field $\vec{E}_0 = E_0\hat{z}$. You will use the separation of variables for the calculation.

a. Show that the boundary conditions for this problem are

i. $V_{in} = V_{out}$ at $r = R$.

ii. $\epsilon \frac{\partial V_{in}}{\partial r} = \epsilon_0 \frac{\partial V_{out}}{\partial r}$ at $r = R$.

iii. $V_{out} \to -E_0 r \cos\theta$ for $r \to \infty$.

b. Given that in a linear dielectric "Gauss's law" applies (i.e. $\vec{\nabla}\cdot\vec{E} = \rho_{free}/\epsilon$, with permittivity $\epsilon = \epsilon_r\epsilon_0$), show that Laplace's equation holds if no free charges are added. Can you apply the separation variables method outside the dielectric? Inside the dielectric?

c. Explain why you will consider solutions of the following forms (a_l, b_l, c_l are constants to be determined):

$$V_{in}(r,\theta) = \sum_{l=0}^{\infty} a_l \left(\frac{r}{R}\right)^l P_l(\cos\theta) \quad \text{for } r \le R.$$

$$V_{out}(r,\theta) = \sum_{l=0}^{\infty} \left[b_l \left(\frac{R}{r}\right)^{l+1} + c_l \left(\frac{r}{R}\right)^l\right] P_l(\cos\theta) \quad \text{for } r \ge R.$$

d. Use the boundary conditions to show that (several long calculations here):

$$V_{in}(r,\theta) = -\frac{3}{\epsilon_r+2} E_0 z \quad \text{for } r \le R.$$

$$V_{out}(r,\theta) = \left[\frac{\epsilon_r-1}{\epsilon_r+2}\frac{R^3}{r^3} - 1\right] E_0 z \quad \text{for } r \ge R.$$

e. Derive expressions for the electric field $\vec{E}_{in}(r,\theta)$ for $r \le R$ and $\vec{E}_{out}(r,\theta)$ for $r \ge R$. Comment on the nature of $\vec{E}_{in}(r,\theta)$.

f. Draw a diagram of the dielectric sphere and the electric field lines in its vicinity (inside and outside).

g. Determine the polarizability $\vec{P}$ of the dielectric inside the sphere (i.e. dipole moment per unit volume) and use this result to calculate the dipole moment $\vec{p}$ of the dielectric sphere.

h. We now suppose that the external electric field $\vec{E}_0$ has a weak gradient to it, which is small enough that your previous work (above) is still valid.

Give an expression for the force on the sphere in terms of the spatial derivatives of $\vec{E}_0$ (i.e. $\frac{\partial}{\partial x}\vec{E}_0$, $\frac{\partial}{\partial y}\vec{E}_0$, $\frac{\partial}{\partial z}\vec{E}_0$).

Is the dielectric sphere a strong field seeker or a weak field seeker?