Monday, April 17, 2023

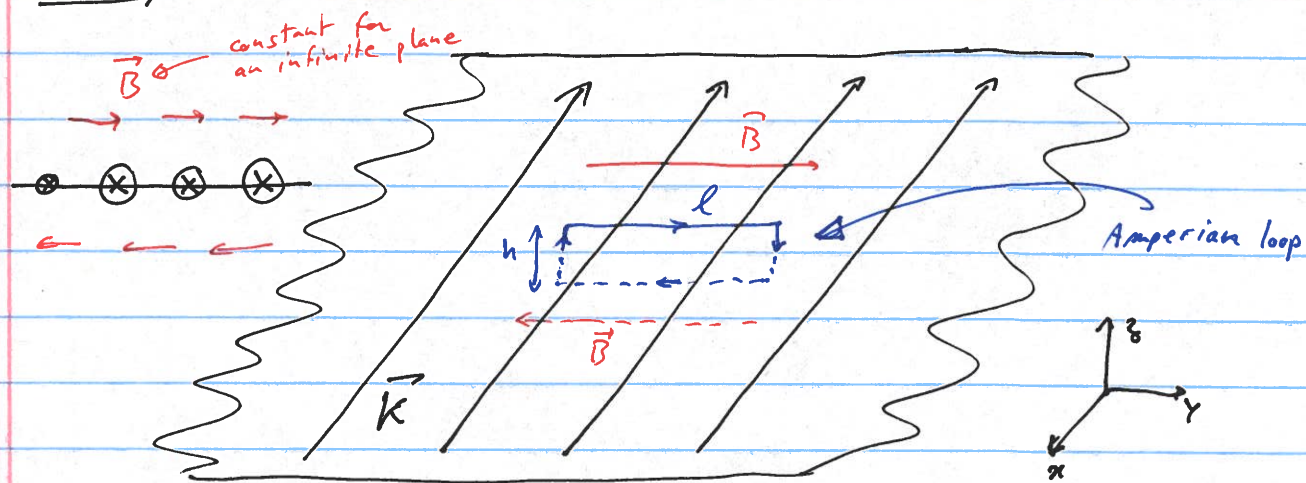
Ampère's Law (continued). [chpt 5.3.3]

$$\boxed{\vec{\nabla}\times\vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r}) \iff \oint_{loop} \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}}$$

magnetic field

current density

Example: Planar Surface Current



$$\oint_{loop} \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

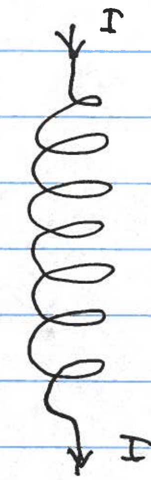
$$\iff \underset{/\!/}{\vec{B}\cdot\vec{l}} + \underbrace{\vec{B}\cdot\vec{h}}_{=0} + \underset{/\!/}{\vec{B}\cdot\vec{l}} + \underbrace{\vec{B}\cdot\vec{h}}_{=0} = \mu_0 K l$$

$\vec{B}\perp\vec{h}$

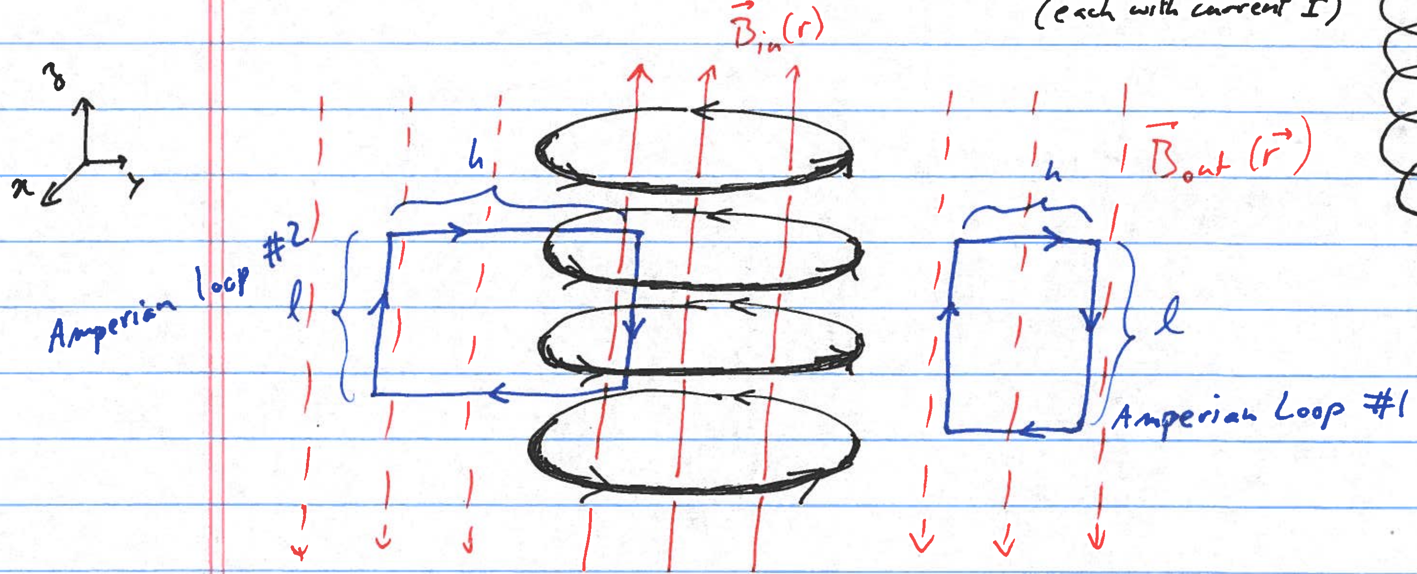
$$\iff 2Bl = \mu_0 K l \iff \boxed{\begin{array}{ll} \vec{B} = \frac{\mu_0 K}{2}\hat{y} & \text{above} \\ \vec{B} = -\frac{\mu_0 K}{2}\hat{y} & \text{below} \end{array}}$$

note: B-field is independent of height. (for an infinite plane)

Example: Ideal Solenoid ("infinitely" long)



↳ Approximate solenoid as a series of independent current loops.
(each with current I)



Amperian loop #1:

$$\oint_{loop\ 1} \vec{B}\cdot d\vec{\ell} = \mu_0 \underbrace{I_{enclosed}}_{=0}$$

$$\Leftrightarrow\ -\vec{B}_{near}\cdot\vec{\ell} + \underbrace{\int \vec{B}\cdot d\vec{h}}_{\perp\ =0} + \vec{B}_{far}\cdot\vec{\ell} + \underbrace{\int \vec{B}\cdot d\vec{h}}_{\perp\ =0} = 0$$

$$\Rightarrow\quad \vec{B}_{near} = \vec{B}_{far} \quad\Leftrightarrow\quad \vec{B}_{out}(r) = \text{constant}$$

Very far from solenoid, we expect/require: $\vec{B}_{out}(r\to\infty) = 0$

$$\Rightarrow\quad \boxed{\vec{B}_{out}(r) = 0}$$

Amperian Loop #2

$$\underbrace{\oint_{loop\ 2} \vec{B}\cdot d\vec{l}}_{-B_{in}\,l + \underbrace{\int \vec{B}\cdot d\vec{l}}_{\perp \,=\, 0} - B_{out}\,l + \underbrace{\int \vec{B}\cdot d\vec{l}}_{\perp \,=\, 0}} = \mu_0 \underbrace{I_{enclosed}}_{NI \,=\, nlI}$$

N: # of turns

n: turns per unit length

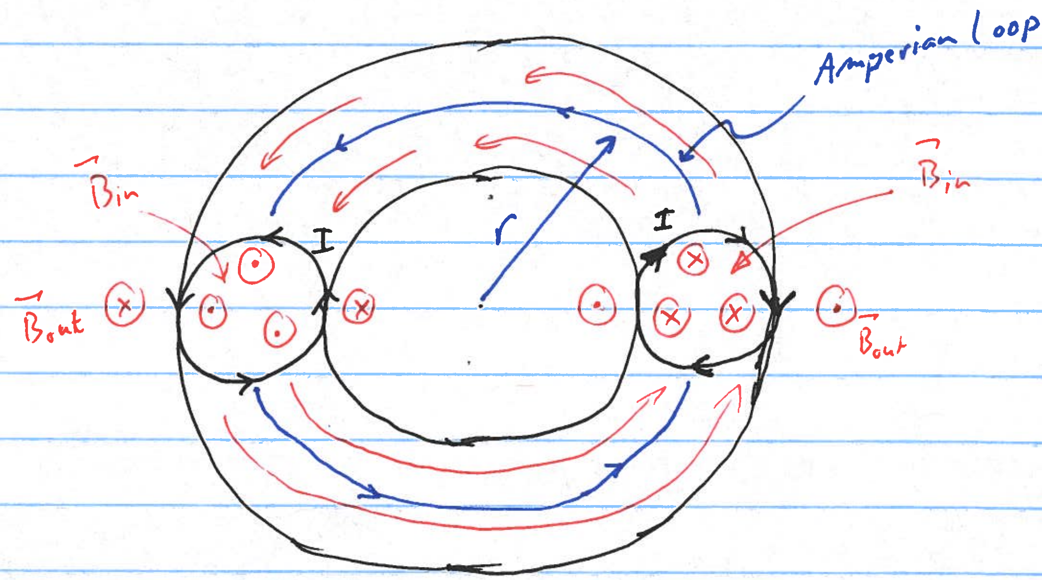
$B_{out} = 0$

$$\Rightarrow \quad -B_{in}\,l = -\mu_0 n l I \quad \Rightarrow \quad \boxed{\begin{aligned} \vec{B}_{in} &= \mu_0 n I \hat{z} \\ \vec{B}_{out} &= 0 \end{aligned}}$$

(no radial dependence!!)

Example: Toroidal coil [Bend solenoid into a doughnut/loop]

↳ Approximate Toroid as a series of identical current loops in the shape of a doughnut / torus.



<u>Amperian loop outside of toroid</u>

$\left\{ \begin{array}{l} r < \text{toroid inner radius} \\ \text{or} \\ r > \text{toroid outer radius} \end{array} \right.$

$$\oint_{\substack{\text{loop}\\\text{out}}} \vec{B} \cdot d\vec{l} = \mu_0 \underbrace{I_{enclosed}}_{=0}$$

$$\Rightarrow \; -B_{out}\, 2\pi r = 0 \quad \Longrightarrow \quad \boxed{\vec{B}_{out} = 0}$$

<u>Amperian loop inside of toroid</u> (toroid inner radius $< r <$ toroid outer radius)

$$\oint_{\substack{\text{loop}\\\text{in}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

total number of turns / loops

$$\Rightarrow \; B_{in}\, 2\pi r = \mu_0 N I$$

$$\Rightarrow \; \boxed{\vec{B}_{in} = \frac{\mu_0 N I}{2\pi r}\, \hat{\phi}}$$

$\Rightarrow$ magnetic field is not constant, but it is completely enclosed inside toroid

(localized magnetic field)

<u>Note</u>:

- Tokamak fusion "reactors" use toroidal magnets (e.g. ITER)
- Nuclei have toroidal currents due to parity violation.
  - ↳ nuclear anapole moment.
  - ↳ contact interaction for electrons.