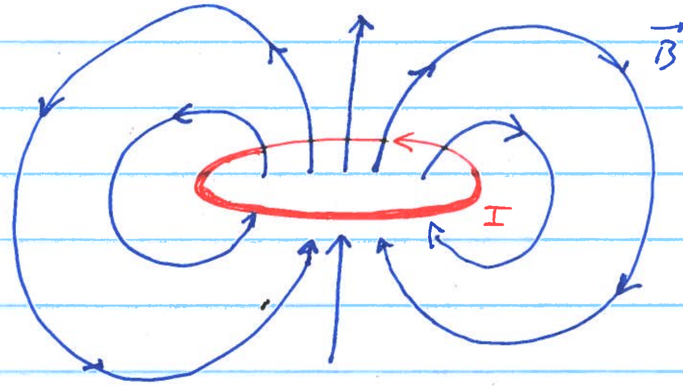
Monday, April 24, 2023

Important magnet coil structures

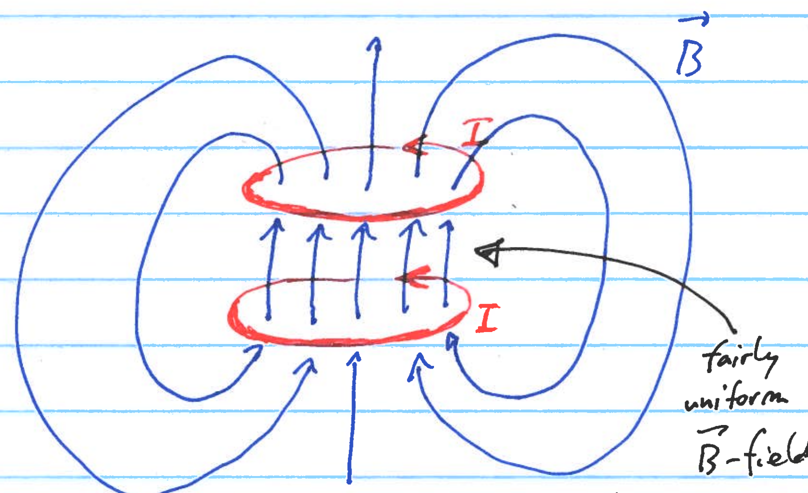
Single coil

similar to an ideal magnetic dipole



Helmholtz coil pair

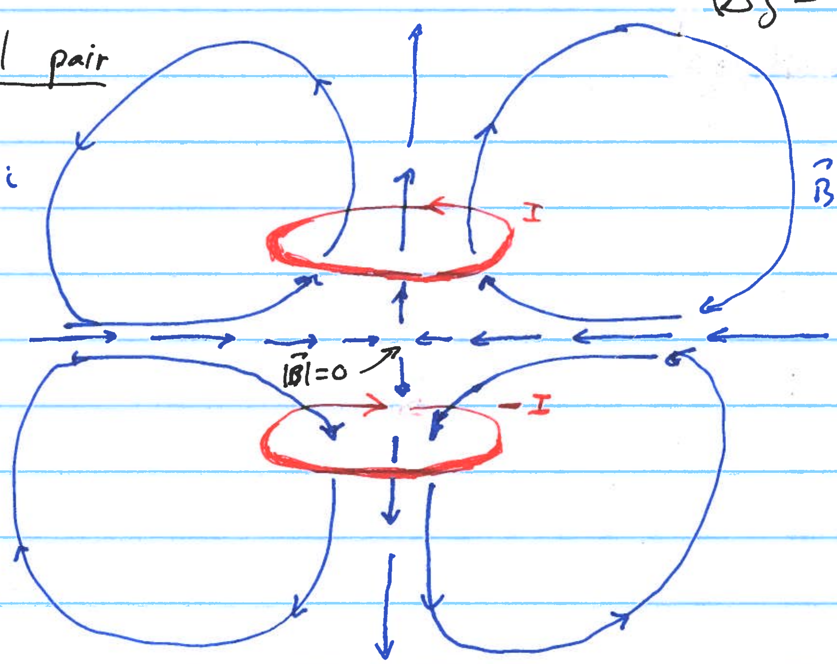
Far away, the B-field is close to an ideal magnetic dipole.



$\Delta z = R$ is optimal.

Anti-Helmholtz coil pair

Quadrupole magnetic field.



Near origin (center), $|\vec{B}|$ is linear.
In fact, $\vec{\nabla}\cdot\vec{B}=0$, so
$\vec{B} = -\beta x\hat{x} - \beta y\hat{y} + 2\beta z\hat{z}$

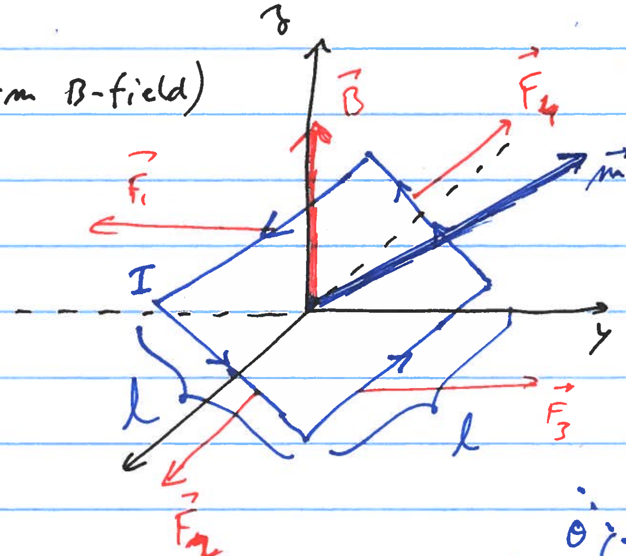
<u>Note:</u> An anti-H coil pair produces a $|\vec{B}|$ minimum, and it can be used to trap "weak field seeking spin states"

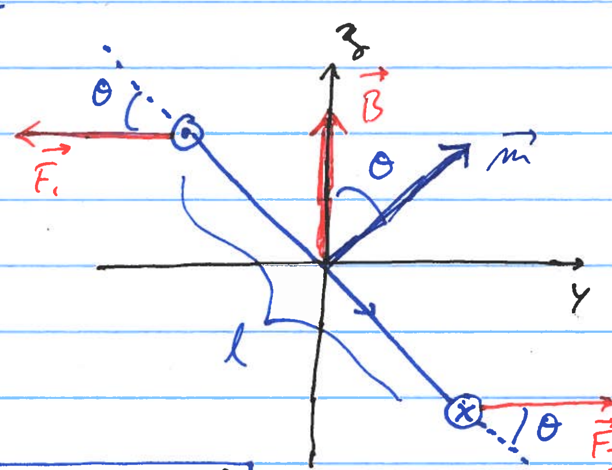
(spin states → magnetic moments)

Forces on Magnetic Dipoles [chpt 6]

Torque
(uniform B-field)

$$\vec{F} = q\vec{v} \times \vec{B}$$
$$= \vec{I} \times \vec{B}\, l$$





The net force is zero, i.e. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$, but there is a net torque from $\vec{F}_1$ & $\vec{F}_3$.

$\Rightarrow$ **Torque tends to align $\vec{m}$ with $\vec{B}$**

$$\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$\vdots$ (2 lines)

$$= \vec{m} \times \vec{B}$$

more generally: $\boxed{\vec{\tau}_{dipole} = \vec{m} \times \vec{B}}$ Torque on a magnetic dipole.

Force

- The force on a magnetic moment in a constant (uniform) B-field is zero.

$$\vec{F} = I \oint (d\vec{\ell} \times \vec{B}) = I\left(\oint d\vec{\ell}\right) \times \vec{B} = 0$$

$\oint d\vec{\ell} = 0$ (closed loop)

- The force on a magnetic dipole in a magnetic field with a gradient is not zero. In fact, the force is:

$$\boxed{\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B})}$$

- Since there is a force, there is also an interaction energy (potential energy):

$$\boxed{U_{\substack{\text{potential}\\ \text{energy}}} = -\vec{m} \cdot \vec{B} = H_{\text{Zeeman}}}$$

note: - If $\vec{m} \parallel \vec{B}$, then $\vec{m}$ is a high field seeker

- If $\vec{m}$ is anti-parallel to $\vec{B}$, then $\vec{m}$ is a low field seeker.
  - ↳ $\vec{m}$ can be trapped by a B-field minimum
  - ↳ ... problem: $\vec{m}$ anti-parallel to $\vec{B}$ is mechanically unstable due to torque
    - ↳ if $\vec{m}$ is spinning like a top (i.e. has angular momentum), then it will be stable.

^{39}K atoms in quadrupole coil magnetic trap

(formed by anti-H coil pair)

temperature $T \sim 100\ \mu K$