Matching conditions for $\vec{B}$ across a surface current $\vec{K}$

Electrostatics

$$\vec{E}_{\substack{\text{surface}\\\text{of charge}}} = \frac{\sigma_s}{2\varepsilon_0}\hat{n} \quad \left(\text{no external } E\text{-fields}\right)$$

External E-field can be present

$$\Delta\vec{E}\Big|_{\text{surface}} = \vec{E}_1\Big|_s - \vec{E}_2\Big|_s = \frac{\sigma_s}{\varepsilon_0}\hat{n}_1$$

$$\vec{E}_{1,\parallel} = \vec{E}_{2,\parallel}$$

$$\Delta\vec{E}\Big|_s \cdot \hat{n}_1 = \frac{\sigma_s}{\varepsilon_0}$$

perpendicular component of $\vec{E}$

Magnetostatics

$$\vec{B}_{\substack{\text{surface}\\\text{current}}} = \frac{\mu_0}{2}\vec{K}\times\hat{n} \quad \left(\text{no external } B\text{-fields}\right)$$

$$\left.\begin{aligned}\Delta\vec{B} &= \vec{B}_1 - \vec{B}_2 = \mu_0\vec{K}\times\hat{n}_1 \\ B_{1,\parallel} - B_{2,\parallel} &= \mu_0 K\end{aligned}\right\} \text{from Ampère's law}$$

"$\parallel$" $\rightarrow$ $\left\{\begin{array}{l}\parallel \text{ to surface} \\ \perp \text{ to } \vec{K}\end{array}\right.$

$$\Delta\vec{B}\Big|_s \cdot \hat{n}_1 = 0 \quad (\text{from } \vec{\nabla}\cdot\vec{B} = 0)$$

Qualitatively:

Electrostatics	Magnetostatics
→ Normal/perpendicular component is affected.	→ perpendicular component is unaffected (i.e. is continuous)
→ parallel component is unaffected (i.e. is continuous)	→ parallel component to surface is affected (but not the one parallel to $\vec{K}$)
V is continuous across boundary	$\vec{A}_\parallel$ is continuous (from Ampère's Law)
	$\vec{A}_\perp$ is continuous in Coulomb gauge
	note: $\frac{\partial\vec{A}}{\partial n}$ is discontinuous

B-fields in Matter [Chapter 6]

Basics of magnetism

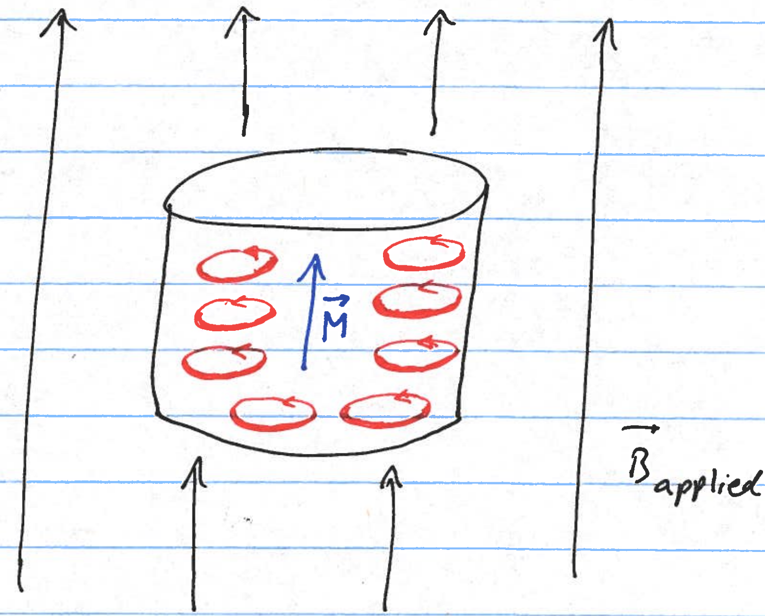
1) Paramagnet

→ $\vec{M} \parallel \vec{B}_{applied,\,local}$.

→ Attracted to high $\vec{B}$.

→ "high field seeker"

(similar to dielectrics $\vec{P} \parallel \vec{E}_{applied,\,local}$)



$\vec{M}$ = "magnetization"

= magnetic moment per unit volume

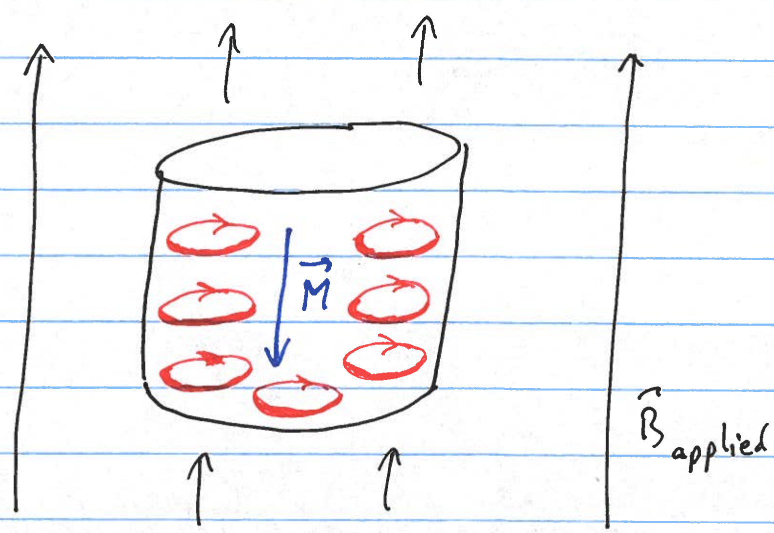
generally from e^- spins, but it can be useful to think of small current loops (classical description).

2) Diamagnet

→ $\vec{M}$ is anti-parallel to $B_{applied,\,local}$

→ Repelled from high $\vec{B}$.

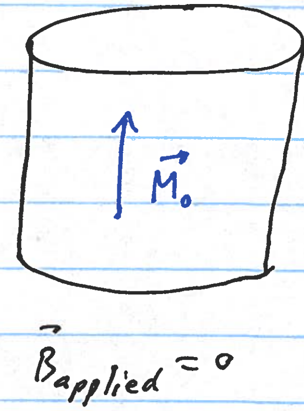
→ "low field seeker"



Ferromagnet

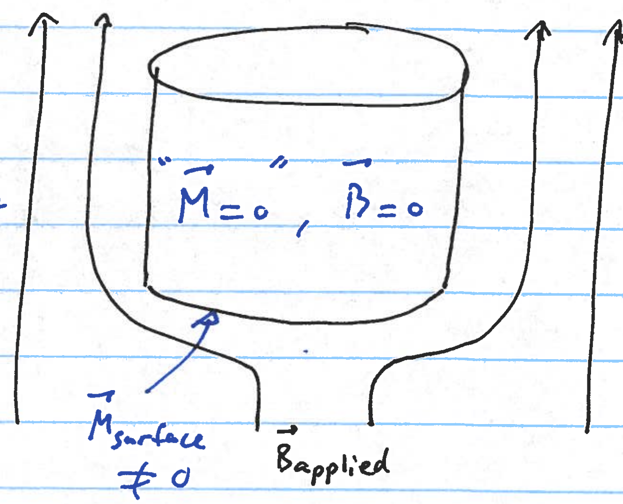
→ $\vec{M}$ depends on history of $\vec{B}_{applied}$.

→ Paramagnet with memory.



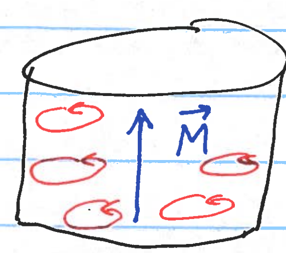
Superconductor

→ sort of like a super/perfect diamagnet



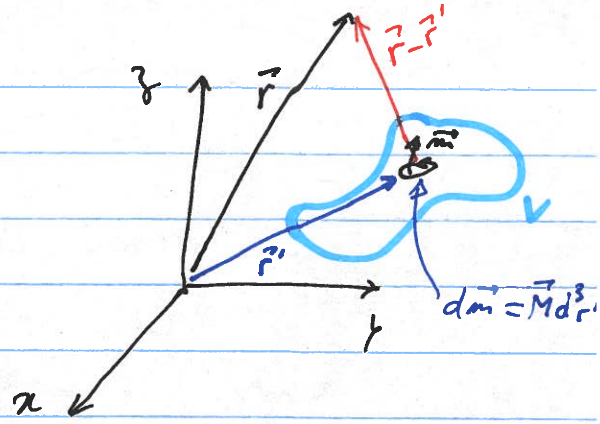
Magnetism in Matter

Basic model: Matter is made up of small magnetic moments, i.e. $\underbrace{\text{small current loops}}_{\text{in reality, electron spins}}$.



$\underbrace{\text{Magnetization}}_{\text{magnetic equivalent of polarization } \vec{P}}$ $\vec{M}$ = total magnetic moment per unit volume

Bound currents



Recall:

$$\vec{A}_{dipole} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \widehat{(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|^2}$$

$$\Rightarrow \vec{A}_M(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{M}(\vec{r}') \times \widehat{(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|^2} d^3r'$$

$$\frac{\widehat{\vec{r}-\vec{r}'}}{|\vec{r}-\vec{r}'|^2} = \vec{\nabla}_{r'} \frac{1}{|\vec{r}-\vec{r}'|}$$

(see also Lecture 17 Monday, April 3)

a few lines of multivariable calculus (use variation on Stokes theorem)

$$\Leftrightarrow \vec{A}_M(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{\nabla} \times \vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} d^3r' + \frac{\mu_0}{4\pi} \int_S \frac{\vec{M}(\vec{r}') \times \hat{n}'}{|\vec{r}-\vec{r}'|} ds'$$

identify $\vec{J}_M = \vec{J}_b$ identify $\vec{K}_M = \vec{K}_b$

recall: $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}') d^3r'}{|\vec{r}-\vec{r}'|}$

Magnetization bound volume current: $\boxed{\vec{J}_b = \vec{\nabla} \times \vec{M} = \vec{J}_M}$

Magnetization bound surface current: $\boxed{\vec{K}_b = \vec{M} \times \hat{n} = \vec{K}_M}$

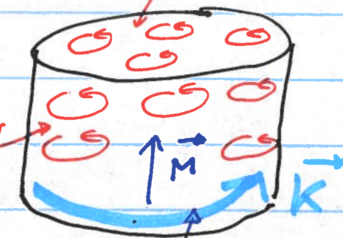
note 1: $\vec{\nabla} \cdot \vec{J}_b = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{M}) = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{J}_b = 0} \Rightarrow \frac{\partial \rho_b}{\partial t} = 0$

bound current is neither created nor destroyed

bound charge is conserved

no bound charge leaves/enters material

note 2: Basic physics



note 3: The surface is where the action is (as in dielectrics).

The Auxiliary Field $\vec{H}$

Total current density: $\vec{J} = \vec{J}_{total} = \vec{J}_b + \underbrace{\vec{J}_{free}}$

$\vec{J}_f$ = current applied by the experimentalist (e.g. Ohm's law)

Ampère's law: $\frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \vec{J}$

$$= \vec{J}_f + \vec{J}_b$$

$$= \vec{J}_f + \vec{\nabla}\times\vec{M}$$

$$\Rightarrow \quad \vec{\nabla}\times\underbrace{\left(\frac{1}{\mu_0}\vec{B}-\vec{M}\right)}_{\vec{H}} = \vec{J}_{free}$$

Auxiliary field $\vec{H}$ (def.): $\boxed{\vec{H} = \frac{1}{\mu_0}\vec{B}-\vec{M}}$ obeys "Ampère's law"

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J}_{free}}$$

$$\Rightarrow \quad \boxed{\oint_{loop} \vec{H}\cdot d\vec{\ell} = I_{free,\,enclosed}}$$

⚠ $\vec{\nabla}\cdot\vec{H} = \frac{1}{\mu_0}\vec{\nabla}\cdot\vec{B} - \vec{\nabla}\cdot\vec{M} \Rightarrow \vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M} \neq 0$

especially on surface where $\vec{K}_b \neq 0$

Linear Magnetic Media (local magnetization is proportional to local field)

$$\boxed{\vec{M} = \chi_m \vec{H}}$$

⚠ Not $\vec{B}$

where χ_m = magnetic susceptibility (dimensionless, no units)

$\chi_m > 0 \Rightarrow$ paramagnetic material

$\chi_m < 0 \Rightarrow$ diamagnetic material

note: χ_m is generally very small.

Examples:

	diamagnetic	paramagnetic
	$\chi_{m_{Cu}} \simeq -10^{-5}$	$\chi_{m_{Al}} \simeq 2\times10^{-5}$
	$\chi_{m_{H_2O}} \simeq -9\times10^{-6} \simeq -0.9\times10^{-5}$	$\chi_{m_{O_2}}$ (gas) $\sim 10^{-6}$
	$\chi_{m_{\text{pyrolytic Carbon}}} \simeq -40\times10^{-5}$	$\chi_{m_{\text{Gadolinium}}} = 0.48$

Ferromagnetic: $\chi_{m_{Fe}} \sim 10^5$

$\chi_{m_{Ni}} \sim 10^2$

Thus, $\vec{B} = \mu_0(\vec{H}+\vec{M}) \Rightarrow \boxed{\vec{B} = \underbrace{\mu_0(1+\chi_m)}\vec{H}}$

μ = magnetic permeability of material $= \mu_0(1+\chi_m)$

$\Rightarrow \boxed{\vec{B} = \mu\vec{H}}$

⚠ $\vec{\nabla}\cdot H_{linear} = \vec{\nabla}\cdot\left(\frac{1}{\mu}\vec{B}\right) = \frac{1}{\mu}\cancelto{0}{\vec{\nabla}\cdot\vec{B}} + \vec{B}\cdot\underbrace{\vec{\nabla}\frac{1}{\mu}}$

$= 0$ inside a uniform material

$\neq 0$ on surface of a material

$\Rightarrow \vec{\nabla}\cdot\vec{H}_{linear} \neq 0$ on surfaces

Comment: For most materials (non-magnetic) $\mu \simeq \mu_0$

↳ most materials have very little effect on magnetic field

↳ most materials are "transparent" to B-field.

(i.e. you can ignore the materials)

[note: you generally generally cannot ignore the presence of dielectric.]

Major Exceptions: mu-metal: $\left.\frac{\mu}{\mu_0}\right|_{\mu\text{-metal}} \sim 10^5$ or higher

used for magnetic shielding

Ferromagnetic materials: $\left.\frac{\mu}{\mu_0}\right|_{Ni,\,Fe,\,Co} \sim 10^2 - 10^5$