

Formula Study Sheet

(i.e. formulas that you should know)

Gradient theorem

$$\int_{\vec{r}_a}^{\vec{r}_b} (\nabla f) \cdot d\vec{l} = f(\vec{r}_b) - f(\vec{r}_a)$$

path P

Divergence Theorem

$$\int_V (\nabla \cdot \vec{F}) d^3r = \oint_{S(V)} \vec{F} \cdot d\vec{s}$$

Stokes's Theorem

$$\int_S (\nabla \times \vec{F}) \cdot d\vec{s} = \oint_{C(S)} \vec{F} \cdot d\vec{\ell}$$

Divergence of $1/r^2$ - point source

$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi\delta^3(\vec{r}) \quad \& \quad \nabla^2 \frac{1}{r} = -4\pi\delta^3(\vec{r})$$

Coulomb's law

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Electric field of a point charge q at \vec{r}'

$$\vec{E}_q = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|^2} (\widehat{r - r'})$$

Electric field of a charge distribution $\rho(\vec{r}')$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^2} (\widehat{r - r'}) d^3r'$$

Potential of a point charge q at \vec{r}'

$$V_q(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}'|}$$

Potential of a charge distribution $\rho(\vec{r}')$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$$

Electric field and potential

$$\vec{E} = -\nabla V \quad \& \quad V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$$

Electric field of a plane of charge

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

Electric field across a plane of charge

$$\Delta E_{\perp} = \frac{\sigma}{\epsilon_0} \quad \& \quad \Delta E_{\parallel} = 0$$

Gauss's law

$$\nabla \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0} \quad \& \quad \oint_S \vec{E} \cdot d\vec{s} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Electrostatic field has no curl

$$\nabla \times \vec{E} = 0$$

Laplace's equation

$$\nabla^2 V(\vec{r}) = 0$$

Poisson's equation

$$\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

Electromagnetic energy

$$U_E = \frac{\epsilon_0}{2} \int \vec{E}^2 d^3r + \frac{1}{2\mu_0} \int \vec{B}^2 d^3r$$

Capacitor of capacitance C

$$C = \frac{Q}{V} \quad \& \quad U_E = \frac{1}{2} CV^2$$

Fourier basis orthogonality relation

$$\int_0^{\pi} \sin(mx) \sin(nx) dx = \frac{\pi}{2} \delta_{mn}$$

Legendre basis orthogonality relation

$$\int_{-1}^1 P_k(x) P_l(x) dx = \frac{2}{2k+1} \delta_{kl}$$

Separation of variables: general solution forms for spherical symmetry

$$V(r, \theta) = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-(n+1)}) P_n(\cos \theta)$$

$$V(r, \theta) = \begin{cases} \sum_{n=0}^{\infty} C_n \left(\frac{r}{R}\right)^n P_n(\cos \theta) & \text{for } r \leq R \\ \sum_{n=0}^{\infty} C_n \left(\frac{R}{r}\right)^{n+1} P_n(\cos \theta) & \text{for } r \geq R \end{cases}$$

Potential of an electric dipole

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

Bound charge and polarization

$$\rho_b(\vec{r}) = -\vec{\nabla} \cdot \vec{P}(\vec{r}) \quad \text{and} \quad \sigma_b(\vec{r}) = \vec{P}(\vec{r}) \cdot \hat{n}$$

Electric displacement field: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

“Gauss’s law” for electric displacement field

$$\vec{\nabla} \cdot \vec{D} = \rho_{free} \quad \& \quad \oint_S \vec{D} \cdot d\vec{s} = q_{free, enclosed}$$

Boundary conditions for dielectrics

$$(\vec{D}_1 - \vec{D}_2)_{\perp} = \sigma_{free}$$

$$(\vec{D}_1 - \vec{D}_2)_{\parallel} = (\vec{P}_1 - \vec{P}_2)_{\parallel}$$

Linear dielectrics

$$\vec{D} = \epsilon \vec{E} \quad \text{with } \epsilon = \epsilon_0(1 + \chi_e)$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{free}}{\epsilon}$$

Magnetostatics

Lorentz force law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Force on a line current: $\vec{F} = I \int d\vec{l} \times \vec{B}$

Biot-Savart law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2} d^3r'$$

Ampère’s law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \& \quad \oint_{loop} \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

No magnetic monopoles law: $\vec{\nabla} \cdot \vec{B} = 0$

Magnetic vector potential: $\vec{B} = \vec{\nabla} \times \vec{A}$

Coulomb gauge definition: $\vec{\nabla} \cdot \vec{A} = 0$

Vector potential in Coulomb gauge

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3r'$$

Magnetic dipole potential (vector)

$$\vec{A}_{dipole}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Magnetic moment (current I , area \vec{a}): $\vec{m} = I\vec{a}$

Bound current and magnetization

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \quad \text{and} \quad \vec{K}_b = \vec{M} \times \hat{n}$$

Auxiliary field: $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$

“Ampère’s law” for the auxiliary field

$$\vec{\nabla} \times \vec{H} = \vec{J}_{free} \quad \& \quad \oint_{loop} \vec{H} \cdot d\vec{l} = I_{free, enclosed}$$

Electrodynamics

Faraday’s law: $\vec{\nabla} \times \vec{E} = -\frac{d}{dt} \vec{B}$

Magnetic flux: $\phi = \oint_{surface-S} \vec{B} \cdot d\vec{s} = LI$

Induced EMF voltage: $\mathcal{E}_{S-border} = -\frac{d}{dt} \phi$

Ohm’s law: $\vec{J} = \sigma \vec{E}$

Energy stored in an inductor L : $U_L = \frac{1}{2} LI^2$

Ampere’s improved law: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{d}{dt} \vec{E}$