PHYS 401: Electricity & Magnetism I
Due date: Wednesday, February 15, 2023

Problem set #3

1) Problem 2.5

2) Problem 2.6

3) Problem 2.7 (solve without Gauss's Law)

4) Problem 2.9

5) Vector calculus review: Archimedes' principle for an object of arbitrary shape
The pressure p as a function of depth in body of water (i.e. lake, ocean, bucket) is given by $p = \rho g h$, where ρ is the density of water (10^3 kg/m^3), g is the local acceleration due to gravity (9.8 m/s^2), and h is the depth below the water surface (h is positive).

The pressure on the surface of a (fully) submerged object manifest itself as a force per unit area $\vec{f}$ that is perpendicular to the surface, i.e. $\vec{f} = -p\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the surface (pointing out).

Task: Use the divergence theorem to show that the water pressure generates a vertically upwards force of $\vec{F} = \rho V g\, \hat{z}$, where V is the volume of the object regardless of its shape ($\hat{z}$ points vertically upwards). In other words, the water pressure produces an upwards buoyancy force that is equal to the mass of the displaced water.